1998

Issues of collective action: common agency, partial cooperation, and clubs

Kevin Jay Siqueira
Iowa State University

Issues of collective action: Common agency, partial cooperation, and clubs

by

Kevin Jay Siqueira

A dissertation submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Major Professor: Todd Sandler

Iowa State University

Ames, Iowa

1998

UMI Number: 9911643

UMI
300 North Zeeb Road
Ann Arbor, MI 48103

Graduate College

Iowa State University

This is to certify that the doctoral dissertation of

Kevin Jay Siqueira

has met the dissertation requirements of Iowa State University

Signature was redacted for privacy.

Major Professor

Signature was redacted for privacy.

For the Major Program

Signature was redacted for privacy.

For the Graduate College

Let him step to the music which he hears, however measured or far away.

– Henry David Thoreau, *On the Duty of Civil Disobedience*

This thesis is dedicated to Todd Sandler and to my parents, to those who have marched ahead and have kept in step with me.

TABLE OF CONTENTS

INTRODUCTION

This thesis has two distinct themes, both revolving around issues of collective action. The first investigates the impact of partial cooperation in a common agency framework and makes up the first two chapters of the dissertation. The second issue to be explored is the relationship between cost sharing and membership size in club models and is the subject of the third chapter. Each issue will be discussed in turn, roughly following the sequence in which they are presented.

The typical approach in exploring topics of collective action and analyzing their results within the context of public goods, usually relies on maintaining the dichotomy between the assumptions of noncooperative behavior on one hand, and complete cooperation among the relevant parties, on the other. The aim of the first two chapters is to fill the gap by exploring the impact of partial cooperation within the context of common agency. In order to do so, we first start by amending the multi-principal, multi-task, single-agent model with hidden action developed by Dixit. This is the main topic of Chapter 1, where effort is devoted to familiarize the reader with the main aspects and characteristics of the model and to set the stage for analyzing the effects of partial cooperation in the subsequent chapter.

Thus in Chapter 1 we review the outcome of noncooperative behavior in a standard common agency model with multiple principals of given types and multiple tasks. There we see and explain the typical result – a weakening of agent incentives with the consequent lowering of agent effort when compared to the fully cooperative case. The cooperative case serves as an important benchmark from which all other results can be measured since it is the best possible outcome, given the problem posed by hidden action. Acknowledging these results, one might expect that at least partial cooperation will improve matters, at least in the direction of moving from a third-best outcome (the standard common agency result) towards a second-best one.

In the first chapter, we are also introduced to a special common agency model that will be extensively used throughout the second chapter. In this version of the model, there are two outcomes, each independently related to the agent's actions, but public to certain types of principals, and private to others. It is also assumed that each action is technically independent of one another so that the agent's cost function is essentially additively separable. These restrictions, in addition to simplifying the presentation, have the added advantage of helping to avoid biasing the case for, or against partial cooperation as a method of improving efficiency within the framework of a common agency.

In Chapter 2 and within the context of the specific model discussed above, partial cooperation means that there is only one type of principal (out of two possible types) likely to form a coalition and act in a unified manner, unilaterally with respect to the other type of principals, who remain unorganized. In this chapter, there are two important cases of concern. The first is a game in which the organized group moves simultaneously with all other unorganized principals when designing the agent's incentive contract. In this game, increased cooperation results in an overall improvement in the agent's effort in both dimensions (of tasks), given that the agent's marginal incentives have been strengthened; however, the cooperating principals are shown to be worse off than had they not formed. Accordingly, there are then no incentives for them to organize.

In the second major scenario, it is assumed that the smallest group has a strategic first-mover advantage over other non-organizing principals. Results under these assumptions show that this strategic advantage not only improves the welfare of this group, but there exists sufficient inducement for them to form a coalition. In addition, agent efforts are strengthened overall, a direct result of a bolstering of the agent's aggregate marginal incentives. The outcome, in fact, is constrained Pareto efficient, better than even third best. Far from worsening matters, partial cooperation coupled with a first-mover advantage, leaves organizing principals better off, in addition to reinforcing agent incentives.

Chapter 2 also includes sections investigating potential ways to alter the rather pessimistic outcomes in Section 2.1. In Section 2.1.1 for example, we examine the impact of payment restrictions The placement of certain payment restrictions is an attempt to limit the indirect transfer of wealth between the two types of principals via the agent. To a certain extent, these restrictions can aid partial cooperation in improving efficiency; thus, it is possible to overturn the previous result of the failure of partial cooperation to improve efficiency in the simultaneous move game. In another section, Section 2.2, we explore two different ways in which a central authority can implement a policy. Though both are a bit more heavy handed when compared to a policy of only imposing restrictions, they try to retain as much of a decentralized orientation as possible. The first policy involves subsidizing the principals' marginal payments to the agent. The second, uses a more direct approach, subsidizing the agent directly. Chapter 2 closes with a discussion concerning the relative merits of intervention.

Issues of collective action and the interdependence of individuals also arise within the environment of congestible public goods. Chapter 3 examines the relationship between cost sharing and membership size within the framework of two club models. The purpose of the chapter is to demonstrate heuristically how impacts on a club's cost structure can lead to changes in membership size by either increasing or decreasing the benefits to cost sharing (among a given number of members). In Section 3.2, we take a well known result from the principal-agent literature – that implementing a given level of effort can be relatively more costly when effort is not observable than when it is – and apply it to a simple club model. It is then shown that this increased cost, the cost of agency, does indeed result in an increased desire for additional members. In the next section, we utilize a slightly more informal modeling approach in order to describe how an outside source of revenue may influence the size of a club's membership base by reducing its need to engage in cost sharing. Results demonstrate that, ceteris paribus, as a consequence of the additional source of revenue applied to covering club costs, membership size can decrease.

In more complicated models, these results may not always hold as they do in the models examined in Chapter 3, but the logic of the connection is at least suggestive. The same caveat holds for the models mainly analyzed in Chapter 2. But nevertheless, the results there indicate an interesting way of approaching problems of collective action, which is through the lens of partial cooperation. Finally, in the conclusion, we quickly summarize the most salient points of the previous three chapters.

CHAPTER 1: COLLECTIVE ACTION AND COMMON AGENCY, PART 1

1.1 Introduction

There are many instances in which actions of individuals have public effects. These effects or externalities exist in circumstances where relationships between the parties that generate these effects and those that are affected by them are formalized either by economic, political, or legal means, or by familial ties. Thus, when asking how these externalities can be internalized or at least ameliorated, it is also important to consider simultaneously to what extent the various participants involved can influence the agent generating the externalities. Institutional design may also play a role, through the goal of trying to improve economic efficiency either by a proper setting of rules and guidelines (which must either be enforceable or within the parties' interests to follow) or by fostering cooperation and bargaining among the parties involved.

A related question is whether a superintendent authority should intercede beyond the aforementioned setting of the rules of the game. Not only does the answer to this question depend on the potential of there being possible and proper venues and channels for cooperation among the parties in question, of whether or not there exists the possibility of collective action and/or contracting, but also on the perceived effectiveness of the proposed policy, its ease of implementation, and its information requirements. This view essentially creates a slightly alternate view towards the standard externality problem that, to a certain extent, goes beyond simple decentralized contracting or Coase bargaining. Even though all parties have a means to exert pressure and influence the agent through either of these methods, there might still be room for the central authority to play a non-passive role in obtaining a better outcome. By allowing the possibility of all having a potential active role to play, we can than assess or at least obtain a better feeling for the likelihood of bringing about the desired changes.

With these features in mind, the problem of common agency represents an appropriate vehicle to explore the impact of conflicting groups of stakeholders or principals on an agent's incentives and the various responses of the principals to the situation they find themselves in. We can also investigate the possibility of intervention by an authority to improve efficiency when the need arises. Although an agent may operate at times under a harsh glare of public or private scrutiny, differences in information between the principals and their agent can fail to disappear or improve. For example, it can be difficult for a stakeholder or a representative of an interest group to be sure whether a public official devotes more time to her concerns or not, let alone to another's. The source of the asymmetric information between principals and the common agent in this case, is one of hidden action. And the important aspect of this problem, and what generally causes deviations from second-best outcomes, is the overall impact of multi-principal competition in simultaneously influencing agent actions and its effect on agent incentives.

The overall goal in using this approach is to provide a better understanding of the difficulties facing various parties concerned and the challenges of collective action, especially when confronting the issue of positive externalities and the appropriate levels of provision in more realistic and decentralized settings. These matters are not irrelevant and are especially pertinent in situations involving multilateral organizations such as the United

Nations or regional and local governments or agencies within multiple jurisdictions and/or with heterogeneous constituents, especially when these type of organizations operate in seemingly opaque environments. Countries acting as principals for example, may have differing goals; some may favor peacekeeping over development. In such circumstances, the attempts of principals to engage and influence the actions of the agent independently can render the agent's incentives ineffective, resulting in inefficient outcomes as compared to the outcome that might occur if all had agreed and were prepared to cooperate fully.

Generally this inefficiency is a result of free riding among principals, but, in a special case to be explored, this may also be due to the fact that common agency can become a conduit within which wealth, albeit indirectly, is transferred between principals. If this is the case, say in multilateral organizations, it is then possible that partial cooperation by a group most likely to solve the free rider problem can, at times, be self-defeating to those groups or if beneficial, then possibly self-defeating from an overall organizational perspective. These reasons may partially explain those events in which gains in efficiency are at times outweighed by one-sided concerns of distribution, when the overall goal of an organization is subverted to serve the parochial interests of one group of members. If the mechanism allowing the transfer of wealth is prohibited, these effects need not occur, and partial cooperation then can move the agency towards improved outcomes. This case may mirror those times when one group of countries or jurisdictions act in concert and whose actions are broadly consistent with the overall goals of the organization. However, in some special circumstances, partial cooperation has the ability to be both beneficial to the organizing group and efficiency enhancing. Thus, it is possible that a group's narrow pursuit of their own interest may not only serve to improve upon certain outcomes, but may also leave other parties at least no worse off than they were before – a "sort of" weakened version of a "win-win" situation.

Although it is not claimed that observable efficiencies or inefficiencies in such organizations or relationships can be solely attributed to success or failure to interdict indirect income transfers among the principals via the agent, it is at least conceivable this issue can exacerbate an already precarious situation. Analogous situations seem to resonate in everyday life. For example, a constituency may withhold support or require compensation (if not similar treatment) from the common agent for actions that benefit others but not itself. The model to be explored is therefore suggestive. But before delving into the full model and matters more fully, it is perhaps advisable at the outset to review some of the main results already existing in the common agency literature.

1.2 General Review of the Common Agency Model

The source of asymmetric information between principals and the common agent can be either a result of hidden information or hidden action. The effect of hidden action is relatively straightforward and although the model to be developed later also concerns the problem of moral hazard, it is perhaps important to review some of the established results. Usually the presence of hidden action – the inability of principals or stakeholders to monitor the agent's action or to enforce a contract based on the agent's actions – causes a weakening of incentives when compared to that of the second-best outcome, that being what is reached under hidden action when the principals are able to cooperate and to contract with the agent on a unified one-to-one basis (see Bernheim and Whinston (1986) and Dixit (1996, 1997)). With a risk-averse agent only caring about the aggregate payment received and the principals all taking the other principals' payments to the agent as given, this result, is due to each principal's incentive to free ride on the contributions of others. That is, each principal is cognizant of the opportunity to derive benefits from the contributions of others while also being acutely aware of the increased costs that one must bear in the form of an additional contribution. Consequently, each principal contributes too little an amount, thus leading to the diminished power of the contract (in terms of the strength of incentives needed to influence the agent's actions).

The problem of common agency with hidden information concerning an agent's characteristics has been investigated by Martimort (1990) and Stole (1990). The results of this problem are slightly more involved than with hidden action. Under similar assumptions as above (except for a risk-neutral agent and the inability of each principal to contract on other variables of direct interest to others, other than its own) hidden information results in outcomes that may fall on either side of the second-best outcome. Whether the result is more or less efficient than the benchmark second-best case depends on whether the contracting variables, the agent's efforts or actions, are substitutes or complements for one another.

In a typical single-principal single-agent problem, the result is a downward distortion in the agent's incentives for the least efficient type of all agents that the principal may contract with. This result also follows through for the multiple-principal single-agent problem if the agent's associated tasks for each principal are contract complements. However in this case, the result is less efficient than second-best outcomes. This follows because the addition of another principal makes it increasingly attractive for the other to distort the agent's effort further downward in response to the other doing the same. If, on the other hand, the contracting variables are substitutes, the desire for each principal to reduce agent effort (again with the aim to reduce rent extraction), in the domain of its concern, is tempered by the consequence that this would have on the agent's incentives. Doing so only makes it more attractive for the agent to substitute away and increase effort in the dimension and direction favorable to the other principal. The overall effect then is a smaller distortion of incentives than even in the second-best case.

A question arises as to what the principals, or even a central authority, can do to improve efficiency, at least in the cases where the probable outcome is worse than second-best efficient Under the typical assumptions of common agency, where increased agent actions are seen to be socially desirable, the solution requires a strengthening of agent incentives. This can be accomplished in several different ways (for example, see Bernheim and Whinston (1986)), but perhaps the easiest to intuit is to allow some degree of cooperation among the principals. In reality this is easier said than done, and yet in some common agency models, the possible added complications are ignored. Increased efficiency can sometimes come at another's expense and therefore it may not pay for some parties to cooperate unless given sufficient additional compensation. This issue and others are mostly addressed later, in Chapter 2, but in order to get there, we first need to introduce and discuss the hidden action multi-principal, multi-task, single-agent model developed by Dixit (1996). Due to the specific nature of the model, the following analysis and some initial results are somewhat familiar although there can be certain important differences when the agent's actions are not completely excludable and when many principals exist within the varying types of the principal population. These differences matter in the presence of collective goods, especially as more stakeholders or constituents find themselves within the range of being affected by and within the sphere of influencing the actions of the agent, while at the same time, also trying to improve on the outcome of the common agency problem.

More specifically, Dixit has demonstrated that it is possible to move from a third-best outcome to a second-best one by restricting principal payments in the dimensions that are not of direct concern to the principals. This was done for the special symmetric case, in which for every task, there is only one principal directly impacted through the single dimension of the agent's effort in a given task. But if there exist more than one principal for every type, the effect of this restriction, made supposedly by mutual agreement, can be to partition the effect of the common agency problem to be limited to remaining within the various groupings as distinguished by type. Efficiency improves since the agent's incentives are strengthened in each dimension, but the result still falls short of securing a second-best outcome since there still exists incentives, for each principal, within every type, to free ride on the payments of others who are of the same type. If a group is able to overcome this problem, which requires not only cash transfers among principals of different types, but also transfers among principals of the same kind, second-best efficiency can be obtained by groups for the domain of their concern. Essentially this result plus an extended discussion of the more general results obtained from the general model are the subjects of the present chapter. The subsequent chapter, as mentioned above, concerns issues of collective action within the context of the common agency problem and is one of the main contributions of the paper. The central issue to be addressed there, is whether there is sufficient motivation for a given type of principal to organize and collectively provide an incentive contract for the agent.

1.3 Model and Initial Results

For simplicity, the agent's tasks or actions are limited to two dimensions. There are also only two types of principals but n^k (k = A, B) of each type so that overall there are $n^A + n^B = n$ principals. Note that when there is only one principal of each type, this will be consistent to Dixit's symmetric m = n case, where the number of tasks (m) equals the number of principals. Matrix notation is not used. This slightly complicates the initial presentation but will make things easier and a bit clearer later on when we investigate potential ways of improving efficiency. Principals are assumed to be risk neutral with preferences specified as

$$U^{ki} = \sum_{j=1}^{2} \Phi_j^{ki} g_j + y^{ki}$$

(k = A, B and $i = 1, \ldots, n^k$) where the subscript refers directly to the output resulting from an agent's action and superscripts index a particular member (i) belonging to a certain type (k) of principals. Hereafter then, when referring to a single principal belonging to a given type, we will refer to that individual as "ki". The additively separable utility function for each principal consists of two elements: a summation of benefits resulting from agent actions where each component, $\Phi_j^{ki} g_j$, is a product of each principal's constant marginal valuation for the output associated with each action of the agent, Φ_j^{ki}, and the output itself, g_j, plus the numeraire good, y^{ki}.

The agent imperfectly controls the observable output upon which the principals can contract on through the technological relationship: $g_j = a_j + \varepsilon_j$ where a_j represents the agent's effort or the level of action for task j, and ε_j is the observational noise inherent in the relationship. ε_j is normally distributed with mean zero, variance σ_{jj} ($j = 1, 2$), and covariance σ_{12}. It is assumed that the risk averse agent has the following CARA utility function, $u(W) = -\exp(-rW)$ with r as the agent's coefficient of risk aversion and the agent's net wealth, W, specified as the difference between the aggregate payment received from the principals, T, and the agent's cost function: $W = T - C(a_1, a_2)$, where C(.) is increasing and strictly convex in both arguments. Note that in this specification, the agent does not obtain any direct benefit from her own actions. This represents a reasonable worst case scenario since the agent's and the principals' preferences are somewhat directly opposed – principals would like to see higher levels of action where the agent desires to exert less. Thus, principals must condition payment in some way related to the agent's effort. If we assume that the principals restrict themselves to using a linear contract when designing an incentive scheme for the agent, aggregate payments to the agent has the following representation: $T = Q + \Sigma_j P_j g_j$, where Q represents the aggregate fixed fee portion of the incentive scheme and P_j, the aggregate variable price per unit for output j. The agent's net wealth then can be written as $W = Q + \Sigma_j P_j g_j - C(a_1, a_2)$. Using the certainty equivalent of wealth form for the agent's utility: $u^{CEW} = E[W] + RP$ where $RP = -\frac{1}{2}(U''/U')var(W)$ is the Arrow-Pratt measure of risk aversion, i.e. the risk premium, we can write

$$u^{CEW} = Q + P_1a_1 + P_2a_2 - (\tfrac{1}{2})r[P_1^2\sigma_{11} + P_2^2\sigma_{22} + 2P_1P_2\sigma_{12}] - C(a_1, a_2).$$

Therefore the solution to the agent's maximization problem is characterized by the first order conditions,

(1) $$P_1 - C_1(a_1, a_2) = 0$$

(1a) $$P_2 - C_2(a_1, a_2) = 0$$

where $C_j = \partial C(.)/\partial a_j$ (j = 1, 2). Writing the solutions to these equations as $a_j = a_j(P_1, P_2)$ for each j, and then utilizing the implicit function theorem, we derive the following comparative statics:

(2) $$\partial a_j/\partial P_j = C_{ii} / D > 0 \quad i, j = 1, 2 \;\; i \neq j$$

(2a) $$\partial a_j/\partial P_i = - C_{ij} / D \quad i, j = 1, 2 \;\; i \neq j$$

with $D \equiv C_{11}C_{22} - C_{12}C_{21} > 0$. Note that if the agent's actions are separable or independent (i.e. the cross partial, C_{ij}, is equal to zero), then $\partial a_j/\partial P_i = 0$ (i, j = 1, 2, i ≠ j). To simplify the upcoming exposition without detracting from the most salient results of the analysis, we assume that the signals are uncorrelated, $\sigma_{12} = 0$, and that the cost function has the following quadratic form,

$$C(a_1, a_2) = (½)a_1^2c_{11} + a_1a_2c_{12} + (½)a_2^2c_{22}\ .$$

It is also assumed that actions can be either independent or substitutes, $c_{12} \geq 0$.[1] Incorporation of the above quadratic form of the agent's cost function then leads to the following alterations of equations (2) and (2a),

(3) $$\partial a_j/\partial P_j = c_{ii} / \Delta > 0 \qquad i, j = 1, 2 \quad i \neq j$$

(3a) $$\partial a_j/\partial P_i = - c_{12} / \Delta \leq 0 \quad i, j = 1, 2 \quad i \neq j$$

with $\Delta \equiv c_{11}c_{22} - c_{12}^2 > 0$.

To ensure the agent's participation, each principal, taking as given the payments to the agent by the other principals, must sufficiently compensate the agent so that her utility is at least as high as she could obtain elsewhere. Thus the individual rationality or participation constraint is

(4) $$Q + P_1a_1 + P_2a_2 - (½)r[P_1^2\sigma_{11} + P_2^2\sigma_{22}] - C(a_1, a_2) \geq 0$$

where the agent's other outside options are exogenous and normalized to be zero. Also note that $Q = \sum_{i\in n^A} Q^{Ai} + \sum_{i\in n^B} Q^{Bi}$ and similarly for each P_j (j = 1, 2).

Since each a_j (j = 1, 2) is not observable or verifiable by the principals, each must ensure that the agent's choice of action, when confronted by his or her payment scheme ($T^{ki} = Q^{ki} + P_1^{ki}g_1 + P_2^{ki}g_2$ or $ET^{ki} = Q^{ki} + P_1^{ki}a_1 + P_2^{ki}a_2$, where E is the expectation operator over the random variables ε_1 and ε_2), is consistent with the agent's own best interests while taking the other payments offered by the principals, as given. That is, each principal must take into consideration the agent's best responses ($a_j(P_1, P_2)$, $j = 1, 2$) when determining the agent's payments. Accordingly we can then summarize the principals' behavior as each principal acting as a Stackelberg leader towards the agent and behaving in a Cournot-Nash fashion vis-à-vis the other principals.

The problem for each principal, ki ($k = A, B$ and $i = 1,\ldots, n^k$), then can be written as

$$\text{Maximize} \qquad EU^{ki} = \sum_{j=1}^{2} \Phi_j^{ki} a_j + y^{ki}$$

$$\text{subject to} \qquad Q + P_1 a_1 + P_2 a_2 - (\tfrac{1}{2}) r[P_1^2 \sigma_{11} + P_2^2 \sigma_{22}] - C(a_1, a_2) \geq 0$$

$$a_j = a_j(P_1, P_2), \; j = 1, 2$$

plus the principal's budget constraint, $Q^{ki} + P_1^{ki} a_1 + P_2^{ki} a_2 + y^{ki} = I^{ki}$, where I^{ki} is exogenous income and assumed to be large enough so that the private good is always strictly positive. Observe that the first constraint is the agent's participation constraint and the second, the agent's incentive compatibility constraint. Solving the budget constraint for y^{ki} and plugging the result into the objective function, we can see that the agent's participation constraint should be binding at the optimum, otherwise the principal can increase utility by either increasing or reducing Q^{ki} without affecting the incentive compatibility constraint. After solving for Q^{ki} using the participation constraint, and plugging the result into the objective function, we obtain the following expression,

$$EU^{ki} = \Phi_1^{ki} a_1 + \Phi_2^{ki} a_2 + I^{ki} + Q^{(-k)} + Q^{k(-i)} + P_1^{(-k)} a_1 + P_1^{k(-i)} a_1 + P_2^{(-k)} a_2 + P_2^{k(-i)} a_2$$
$$- (\tfrac{1}{2}) r[P_1^2 \sigma_{11} + P_2^2 \sigma_{22}] - C(a_1, a_2)$$

where $Q^{(-k)}$ equals the fixed fees paid by all principals of the other type and $Q^{k(-i)}$ represents the fixed fees paid by all principals of the same type as principal i, but excluding principal i's contribution. A similar interpretation holds for the variable payments made to the agent by all principals other than principal ki. The problem then is to

$$\text{Maximize}\ EU^{ki} = \Phi_1^{ki} a_1 + \Phi_2^{ki} a_2 + I^{ki} + Q^{(-k)} + Q^{k(-i)} + P_1^{(-k)} a_1 + P_1^{k(-i)} a_1 + P_2^{(-k)} a_2 + P_2^{k(-i)} a_2$$

$$- (\tfrac{1}{2}) r [P_1^2 \sigma_{11} + P_2^2 \sigma_{22}] - C(a_1, a_2)$$

$$\text{subject to} \qquad a_j = a_j(P_1, P_2),\ j = 1, 2\ .$$

After incorporation of the incentive compatibility constraints (or the agent's response functions), we can then proceed with the optimization problem by taking derivatives with respect to P_j^{ki} (j = 1, 2). Using the agent's first-order conditions, equations (3) and (3a) and after some slight manipulations, we obtain the following first-order conditions[2]

(5) $$P_1^{ki}: \quad \Phi_1^{ki} c_{22} - \Phi_2^{ki} c_{12} - P_1^{ki} c_{22} + P_2^{ki} c_{12} - r\sigma_{11} \Delta P_1 = 0$$

(5a) $$P_2^{ki}: \quad -\Phi_1^{ki} c_{12} + \Phi_2^{ki} c_{11} + P_1^{ki} c_{12} - P_2^{ki} c_{11} - r\sigma_{22} \Delta P_2 = 0\ .$$

Then summing these equations over all members of type A (i = 1,..., n^A) gives

(6) $$P_1^{A}: \quad \Phi_1^{A} c_{22} - \Phi_2^{A} c_{12} - P_1^{A} c_{22} + P_2^{A} c_{12} - n^A r\sigma_{11} \Delta P_1 = 0$$

(6a) $$P_2^{A}: \quad -\Phi_1^{A} c_{12} + \Phi_2^{A} c_{11} + P_1^{A} c_{12} - P_2^{A} c_{11} - n^A r\sigma_{22} \Delta P_2 = 0$$

where $P_1^A = \sum P_1^{Ai}$, $P_2^A = \sum P_2^{Ai}$, $\Phi_1^A = \sum \Phi_1^{Ai}$, and $\Phi_2^A = \sum \Phi_2^{Ai}$. And similarly for type B (i = 1,..., n^B):

(7) $$P_1^{B}: \quad \Phi_1^{B} c_{22} - \Phi_2^{B} c_{12} - P_1^{B} c_{22} + P_2^{B} c_{12} - n^B r\sigma_{11} \Delta P_1 = 0$$

(7a) $$P_2^{B}: \quad -\Phi_1^{B} c_{12} + \Phi_2^{B} c_{11} + P_1^{B} c_{12} - P_2^{B} c_{11} - n^B r\sigma_{22} \Delta P_2 = 0$$

with $P_1^B = \sum P_1^{Bi}$, $P_2^B = \sum P_2^{Bi}$, $\Phi_1^B = \sum \Phi_1^{Bi}$, and $\Phi_2^B = \sum \Phi_2^{Bi}$. Finally summing equations (6) and (7) together and (6a) together with (7a), results in

(8) $\quad P_1: \quad \Phi_1 c_{22} - \Phi_2 c_{12} - P_1 c_{22} + P_2 c_{12} - nr\sigma_{11}\Delta P_1 = 0$

(8a) $\quad P_2: \quad -\Phi_1 c_{12} + \Phi_2 c_{11} + P_1 c_{12} - P_2 c_{11} - nr\sigma_{22}\Delta P_2 = 0$.

And also for completeness, $P_1 = P_1^A + P_1^B$, $P_2 = P_2^A + P_2^B$, $\Phi_1 = \Phi_1^A + \Phi_1^B$, and $\Phi_2 = \Phi_2^A + \Phi_2^B$

At this point, we now can solve for the agent's aggregate variable fees, paid by the principals, using equations (8) and (8a),

(9) $\quad P_1 = [\Phi_1(1 + nr\sigma_{22}c_{22}) - \Phi_2 nr\sigma_{22}c_{12}](1/\Psi)$

(9a) $\quad P_2 = [\Phi_2(1 + nr\sigma_{11}c_{11}) - \Phi_1 nr\sigma_{11}c_{12}](1/\Psi)$

where $\Psi \equiv 1 + nr(\sigma_{11}c_{11} + \sigma_{22}c_{22} + nr\sigma_{11}\sigma_{22}\Delta)$. In order for the agent's first-order conditions to hold, both (9) and (9a) must be positive. Sufficient conditions for these are $[\Phi_1 c_{22} - \Phi_2 c_{12}] > -(\Phi_1/nr\sigma_{22})$ and $[\Phi_2 c_{11} - \Phi_1 c_{12}] > -(\Phi_2/nr\sigma_{11})$. Note that in the presence of economies of scope, $c_{12} < 0$, these conditions are easily satisfied and in addition, serve to strengthen the agent's aggregate variable fees. Using the expressions for P_1 and P_2, one can now go back and solve for most of the remaining variables and functions of interest (P_1^{Ai} , P_1^A , a_1, etc…). The exceptions being Q^A, Q^B, and each Q^{ki}, since all are indeterminate.

To interpret these results, it is instructive to compare these to the first-best case under perfect observability or when the agent is risk neutral. For the latter case, $r = 0 \Rightarrow P_1^{fb} = \Phi_1$ and $P_2^{fb} = \Phi_2$, the superscript "fb" signifying first best. This result holds with or without principal cooperation. In this case, each aggregate marginal payment to the agent is exactly equal to the principals' aggregate valuation of the benefit or output resulting from the relevant action taken by the agent. Comparing these expressions to equations (9) and (9a) we see that each aggregate payment to the agent is set equal to a weighted sum of not only the aggregate valuation of the benefit itself, but also to the difference between the same aggregate valuation and that of the other, each weighted by terms associated with agent risk aversion and cost parameters and the total number of principals. This is then divided by the term Ψ. It is possible to show that $P_1^{fb} > P_1$ and $P_2^{fb} > P_2$, that is, the aggregate marginal contributions to the agent are greater under conditions that produce first-best outcomes. In addition, it can also be shown that $a_j^{fb} > a_j$ (j = 1, 2) or that the agent exerts less than first-best effort under conditions of common agency with asymmetric information.[3]

One aspect behind the resulting inefficiency of the common agency when compared to the first-best case (for now we will ignore the other, the problem of free riding, until we compare equations (9) and (9a) to outcomes that are second best) is that in the latter situation when the agent is risk neutral, there is no need to trade off economic efficiency with risk sharing. Principals have no reason to distort or weaken marginal incentives in order to appease the agent. This allows the aggregate surplus between the parties to be as large as possible since it is within both their interests. Presumably part of this surplus is rebated back to the principals with negative fixed fees. Alternatively, when there is perfect observability, the principals are able to contract directly on effort, and therefore completely able to sidestep the problem of conditioning contracts under uncertainty (or rather, asymmetric information).

Next we compare (9) and (9a) to those expressions derived under conditions of second best. Usually referred to as the united principals case, principals, with the aid of implied inter-principal cash transfers, agree to maximize the sum of their expected utility with respect to P_1, P_2, and Q, subject to the agent's participation and incentive compatibility constraints and the principals' resource constraint. The only difference between the noncooperative and cooperative solutions derived from the marginal conditions is that for the latter, the expressions for P_1^{sb} and P_2^{sb} all show a conspicuous lack of "n-terms". These expressions are shown in (10) and (10a):

(10) $$P_1^{sb} = [\Phi_1(1 + r\sigma_{22}c_{22}) - \Phi_2 r\sigma_{22}c_{12}](1/\Psi^{sb})$$

(10a) $$P_2^{sb} = [\Phi_2(1 + r\sigma_{11}c_{11}) - \Phi_1 r\sigma_{11}c_{12}](1/\Psi^{sb})$$

and $\Psi^{sb} \equiv 1 + r(\sigma_{11}c_{11} + \sigma_{22}c_{22} + r\sigma_{11}\sigma_{22}\Delta)$. Thus the only difference, to paraphrase Dixit (1997), is that in the former case where the principals act separately, the risk-aversion parameters are all magnified by a factor n. And as can be seen, this not only results in a weakening of the marginal payments to the agent ($P_1^{sb} > P_1$ and $P_2^{sb} > P_2$) but also a weakening in the agent's efforts ($a_1^{sb} > a_1$ and $a_2^{sb} > a_2$).

In order to understand the resulting inefficiency in the common agency problem and the role of free riding, we rewrite equations (5) and (5a) as

(11) $$P_1^{ki}: \quad [\Phi_1^{ki} - P_1^{ki}](\partial a_1/\partial P_1) + [\Phi_2^{ki} - P_2^{ki}](\partial a_2/P_1) - \partial RP/\partial P_1 = 0$$

(11a) $$P_2^{ki}: \quad [\Phi_2^{ki} - P_2^{ki}](\partial a_2/\partial P_2) + [\Phi_1^{ki} - P_1^{ki}](\partial a_1/\partial P_2) - \partial RP/\partial P_2 = 0$$

where $RP = -(½)r[P_1^2\sigma_{11} + P_2^2\sigma_{22}]$. From these conditions, it is then easy to see that each principal considers the impact of their marginal choices on the agent's actions and risk premium. Nevertheless he fails to take into account the flow of net benefits to the other principals resulting from changes in the agent's actions. Since these effects are in the form of positive externalities to the remaining principals, each principal provides the agent with incentives that are too weak. To see this another way but what amounts to the same thing, each principal treats the marginal effect on the agent's risk premium resulting from his increased payment to the agent as a private cost whereas in effect it is a cost faced by all principals. So in weighing his marginal benefits to his marginal costs, he gives too much weight, from the standpoint of efficiency, to the costs he faces. Either way one looks at it, principals have incentives to free ride.

As an aside, note in equations (11) and (11a), the presence of the respective terms: $[\Phi_2^{ki} - P_2^{ki}](\partial a_2 / \partial P_1)$ and $[\Phi_1^{ki} - P_1^{ki}](\partial a_1 / \partial P_2)$. Each of these terms enters negatively into their respective equations and represents an effect that each principal treats as an added cost (or as a negative marginal benefit): the agent's substitution of effort away from the other activity as payment is increased to reward effort in the direction of the observed outcome of concern.[4]

To reduce the source of conflict and the potential for free riding, assume the following: that $c_{12} = 0$, and that type A principals only directly care about the output or signal generated from or related to a_1 ($\Phi_1^{Ai} > 0, \Phi_2^{Ai} = 0$) and that only type B principals are directly concerned with output generated from a_2 ($(\Phi_1^{Bi} = 0, \Phi_2^{Bi} > 0)$. These restrictions result in the following first-order conditions[5] for members of types A and B,

(12) $\qquad P_1^{Ai}: \quad \Phi_1^{Ai} - P_1^{Ai} - r\sigma_{11}c_{11}P_1 = 0$

(12a) $\qquad P_2^{Ai}: \quad -P_2^{Ai} - r\sigma_{22}c_{22}P_2 = 0$

(12b) $\qquad P_1^{Bi}: \quad -P_1^{Bi} - r\sigma_{11}c_{11}P_1 = 0$

(12c) $\qquad P_2^{Bi}: \quad \Phi_2^{Bi} - P_2^{Bi} - r\sigma_{22}c_{22}P_2 = 0.$

Note that although aggregate variable payments to the agent must be positive, individual payments in certain dimensions can be zero or negative. The intuition for this outcome is that even, if say, a principal of type A is not directly affected by the agent's choice of action, a_2, an incentive still exists for the principal to influence the agent's actions. By setting $P_2^{Ai} < 0$, the principal penalizes the agent for higher realized outcomes for output that are not directly of concern to him. This in turn requires higher payments from type B principals to offset these negative incentives and so, in effect, these fines paid to type A principals by the agent are financed by type B principals. Since the same holds true for P_1^{Bi}, the impact is a lowering of incentives overall (Dixit 1996, p. 169). These effects, all taken together, will also play an important role in the subsequent chapter when we look at the possibility of collective action. In addition, note that in the case where $\Phi_1^{Ai} > 0$, $\Phi_2^{Ai} = 0$, $\Phi_1^{Bi} = 0$, $\Phi_2^{Bi} > 0$, but $c_{12} > 0$, not only does the negative marginal payment, $P_2^{Ai} < 0$, discourage the agent from exerting effort in the direction of task two, but because of the substitution effect, $\partial a_2/\partial P_1$, the agent is also encouraged to expend more effort in the dimension of task one, all things being equal.[6]

For completeness and future reference, summing each of the above equations, (12)-(12c), over all principals of a particular type, and then by types for each k (k = A, B), results in

(13) $$P_1 = \frac{\Phi_1^A}{1 + nr\sigma_{11}c_{11}}$$

(13a) $$P_2 = \frac{\Phi_2^B}{1 + nr\sigma_{22}c_{22}}.$$

Using these expressions and the above four first-order conditions, we can derive the group levels of the marginal payments to the agent after summing over all i members for each type of principal:

(14) $$P_1^A = \frac{\Phi_1^A(1+n^B r\sigma_{11}c_{11})}{1+nr\sigma_{11}c_{11}}$$

(14a) $$P_2^A = -\frac{\Phi_2^B n^A r\sigma_{22}c_{22}}{1+nr\sigma_{22}c_{22}}$$

(14b) $$P_1^B = -\frac{\Phi_1^A n^B r\sigma_{11}c_{11}}{1+nr\sigma_{11}c_{11}}$$

(14c) $$P_2^B = \frac{\Phi_2^B(1+n^A r\sigma_{22}c_{22})}{1+nr\sigma_{22}c_{22}}.$$

And from the agent's first-order conditions,

(15) $$a_1 = (1/c_{11})P_1 = (1/c_{11})\frac{\Phi_1^A}{1+nr\sigma_{11}c_{11}}$$

(15a) $$a_2 = (1/c_{22})P_2 = (1/c_{22})\frac{\Phi_2^B}{1+nr\sigma_{22}c_{22}}.$$

From the expressions listed in (13)-(13a) and (14)-(14c), it is possible to see the effect of principals of one type punishing the agent for realizations of outcomes that are not of direct concern to them. These effects force the other principals as a group to pay higher variable fees than the final (aggregate) variable for a given dimension so as to offset the negative impacts of punishment by the principals of the other type. For example, $P_1^A > P_1$ in order to offset the effects of P_1^B.[7]

Given the simplifying assumptions and the agent's participation constraint at the optimum, we can try to gauge the impact of common agency on the agent's fixed fee. Solving the participation constraint for Q gives

$$Q = -P_1a_1 - P_2a_2 + (\tfrac{1}{2})r[P_1^2\sigma_{11} + P_2^2\sigma_{22}] + C(a_1, a_2).$$

Incorporating the incentive compatibility constraints and differentiating (without loss of generality) by P_1, results in

$$\partial Q/\partial P_1 = -a_1 - P_1(\partial a_1/\partial P_1) + r\sigma_{11}P_1 + C_1(\partial a_1/\partial P_1)\,.$$

Using the agent's first-order conditions and the expression for a_1, the above expression can be rewritten as

$$\partial Q/\partial P_1 = P_1[r\sigma_{11} - (1/c_{11})]$$

or

$$\text{sign}(\partial Q/\partial P_1) = \text{sign}[r\sigma_{11} - (1/c_{11})]\,.$$

So the response of Q to changes in P_1 can be positively related, independent of, or negatively related depending on whether $r\sigma_{11}$ is greater than, equal to, or less than $1/c_{11}$. For the rest of this paper it will be assumed that $\partial Q/\partial P_1 > 0$, meaning that a unit change in P_1 has a greater impact on the agent's risk premium than it does on the agent's response in effort, i.e. $\partial^2 RP/\partial P_1^2 = r\sigma_{11} > 1/c_{11} = \partial a_1/\partial P_1$. Thus, as variable payments decrease as a result of the common agency problem, the fixed portion of the agent's fees declines as well.

Finally by listing solutions derived under the assumption of cooperation, it is easy to see the suboptimality of the above outcome relative to the second-best case:

(16) $$P_1^{sb} = \frac{\Phi_1^A}{1 + r\sigma_{11}c_{11}}$$

(16a) $$P_2^{sb} = \frac{\Phi_2^B}{1 + r\sigma_{22}c_{22}}$$

(17) $$a_1^{sb} = (1/c_{11})P_1^{sb} = (1/c_{11})\frac{\Phi_1^A}{1 + r\sigma_{11}c_{11}}$$

$$a_2^{sb} = (1/c_{22})P_2^{sb} = (1/c_{22})\frac{\Phi_2^B}{1 + r\sigma_{22}c_{22}}. \quad (17a)$$

That is, the effects of multi-principal competition and free riding remain and so therefore overall, $P_1^{sb} > P_1$, $a_1^{sb} > a_1$, and so on.

Under conditions of second best, it is assumed that the appropriate cash transfers among principals take place so as to internalize the externalities resulting from the combination of free riding and acting separately when determining the agent's incentives. However, even if one assumed the presence of at least, partial collective action in an attempt to strengthen agent incentives, results will show, starting in the next section and chapter two, that even this result may be unlikely unless one assumes cash transfers taking place not only among the collectivized principals, but also between this group and the unorganized principals. If such an event is considered unlikely, then other means of improving efficiency must be sought.

1.4 Further Discussion

Under standard common agency assumptions, principal cooperation or any form of side contracting between them is assumed not to take place either due to problems of coordination or because such behavior or arrangements are frowned upon by the authorities, possibly because of anti-trust concerns. These initial assumptions serve as a useful starting point and can be justified in settings where principals are numerous if not varying widely in tastes and income or in market settings consisting of small firms selling similar or slightly differentiated products and the common agent, a marketing agency. This latter case has been investigated by Bernheim and Whinston (1985).[8] In scenarios concerning the production of collective goods or services, whether by an international multilateral agency or by a national or even a local governmental agency, the same assumption concerning the principals' inability to cooperate can also be pertinent. For example, individual citizens of a nation may feel that, whether organized or not, their actions would have little impact in influencing the decisions of a federal agency. Or if such actions could, the derived benefits may still not be large enough to outweigh or justify any costs incurred in influencing the behavior of the agency.

As mentioned previously, actions by an agent or more likely, a public official or agency has the potential of having its effects felt across many individuals and organizations, of varying and similar interests. But since individuals of similar interests might find themselves under like circumstances and given that such homogeneity can sometimes ease the way for cooperation, our attention should not be restricted in scope so as to preclude the possibility for cooperation or only reserve it for the times when it is needed as a benchmark from which to measure other results by. Furthermore, by allowing many principals to be of a given type permits us to vary the scale of which cooperation may make itself evident. This also provides us the opportunity to explore interesting issues in more realistic settings. For example, using the restricted model from above, we can investigate the effects of inter- and intra-type principal cooperation when an agent's actions provide localized public services or goods without spillovers. The same model can also serve as a simplified but stark depiction of a multilateral agency operating under asymmetric information when the agent's actions only benefit certain segments of the membership.

Within the context of our restricted model, when there are no spillovers across principal type, i.e., when an agent's output is not of direct concern to certain types of principals, it was shown that there still remained incentives for principals to influence the agent's actions in these dimensions. Specifically we saw that $P_2^{Ai} < 0$ and $P_1^{Bi} < 0$. We also saw how this forced principals as a group to pay higher marginal incentives to the agent even though the overall effect was diminished in aggregate: $P_1^A > P_1$ and $P_2^B > P_2$. Still, if it were possible for principals to mutually agree upon restricting their ability to contract on these variables in their dealings with the agent, as proposed by Dixit (1996, 1997), this is not sufficient to replicate the second-best solution. This recommendation works for the case when there is only one principal of each type as shown by Dixit, but not when there are more than one principal for each kind. That is, efficiency will improve but since there remain incentives for principals of a given type to free ride, the outcome still falls short of second best. Therefore, the only effect of restricting payments in these directions for certain types of principals is to partition the effects of the common agency problem so that they are limited to specific groups of principals. It is as if members of each type of principal as a group, but acting separately, had their own common agent.

This can be readily seen using equations (12)-(12c). If by mutual agreement, presumably aided by cash transfers among principals – which would occur for example, if a reshifting of the burden of the fixed fee to be paid to the agent is not sufficient enough to do the job – principals agree to set $P_2^{Ai} = P_1^{Bi} = 0$, the first-order conditions reduce to

$$(18) \qquad P_1^{Ai}: \quad \Phi_1^{Ai} - P_1^{Ai} - r\sigma_{11}c_{11}P_1^A = 0$$

$$(18a) \qquad P_2^{Bi}: \quad \Phi_2^{Bi} - P_2^{Bi} - r\sigma_{22}c_{22}P_2^B = 0.$$

Then as before, summing equation (18) over all $i \in n^A$ and (18a) for all $i \in n^B$, and solving each respectively for $P_1^{A,RU}$ and $P_2^{B,RU}$ results in

(19) $$P_1^{RU} = P_1^{A,RU} = \frac{\Phi_1^A}{1 + n^A r\sigma_{11}c_{11}}$$

(19a) $$P_2^{RU} = P_2^{B,RU} = \frac{\Phi_2^B}{1 + n^B r\sigma_{22}c_{22}}$$

where the "RU" of the superscript refers to the characterization of the solution we investigate – that principals restrict their payments in certain dimensions but remain unorganized, separately making payments to the agent in the dimension of direct concern to them. Using these results to derive expressions for the agent's optimal levels of effort gives

(20) $$a_1^{RU} = (1/c_{11})\frac{\Phi_1^A}{1 + n^A r\sigma_{11}c_{11}}$$

(20a) $$a_2^{RU} = (1/c_{22})\frac{\Phi_2^B}{1 + n^B r\sigma_{22}c_{22}}\ .$$

Equations (19), (19a), (20), and (20a) show that, under this agreement, principals are able to increase the agent's level of effort as compared to the original third-best outcome under common agency. As can be seen, these solutions still fall short of second best. That is, $P_1 < P_1^{RU} < P_1^{sb}$ and $P_2 < P_2^{RU} < P_2^{sb}$ so that $a_1 < a_1^{RU} < a_1^{sb}$ and $a_2 < a_2^{RU} < a_2^{sb}$. Moreover under common agency, type A principals, as a group pay less in terms of the marginal incentive payment to be paid to the agent for g_1 outcomes, but receive less payment for output realizations that are not of direct concern to them. This also holds for type B principals so that overall we have

$$P_1^A > P_1^{A,RU} \qquad P_2^B > P_2^{B,RU}$$

$$\left|P_2^A\right| > \left|P_2^{A,RU}\right| = 0 \qquad \left|P_1^B\right| > \left|P_1^{B,RU}\right| = 0\ .$$

In terms of the marginal conditions, this means that aggregate utility for each type of principal is likely to be ambiguous and therefore successful bargaining over cash transfers among principals, in addition to the determining of the distribution of fixed payments to be made to the agent (among principals), is needed to ensure participation for all concerned.

It should be noted here that expressions (19) and (19a) can also be viewed as Samuelson conditions for two collective goods. Given our present working assumptions, first best entails $\Phi_1^A = P_1^{fb} = C_1$ and $\Phi_2^B = P_2^{fb} = C_2$, or in words, that the summation of marginal benefits resulting from an additional unit of action taken by the agent be equated to its marginal cost. From (19) and (19a) we see, in each case, that only a fraction of the aggregate benefits, say for a particular community of similar types, is equated to marginal cost. This results in lower levels of action taking by the agent and hence lower expected output.

Of related interest, it is worth comparing the above results, say equations (19) and (20), with that of the standard voluntary provision of a public good model with quasilinear utility (for example, see Cornes and Sandler, 1996, Chapter 6). The standard result, with Cournot-Nash behavior but without the problem of asymmetric information, is also suboptimal in relation to the cooperative case. The public good, or in the context of our present discussion, the agent's action, is underprovided. However the Cournot-Nash equilibrium of that game is independent of the number of contributors, where this is not the case for common agency with hidden action as can be seen in the aforementioned equations. It also can be shown in the latter case, that as the number of each type of principal increases, the agent's levels of effort decline further. In addition, the suboptimal outcome of the common agency apparently follows directly from the problem of asymmetric information and the noncooperative behavior of the principals. However, once the problem of asymmetric information is eliminated between the principals and the agent under common agency, the provision of agent services are no longer suboptimal. The difference in this result over the typical voluntary provision of the public good model, despite the fact that noncooperative behavior is postulated for both, is each principal's ability to specify a contract and to contract for specific levels of performance by the agent.[9] There is no similar ability or comparative option, explicitly or implicitly, under the standard assumptions of the voluntary provision model.[10]

Finally returning to our discussion concerning ways to improve upon third-best outcomes within the context of our present assumptions, it is not too hard to imagine that with a bit more cooperation, matters can be further improved upon. Suppose for example, that in addition to imposing restrictions on certain types of payments to be made to the agent, principals of similar tastes and income are able to overcome the free-rider problem and organize into specialized groups representing their own interests. Although this process is in itself hard to model, we can speculate that this level of cooperation may have been fostered by similarity of tastes and income among principals or a result of their geographical proximity to one another (which admittedly could have been the initial cause for similarity of the principals). Either way, assume that principals of the same type agree to maximize the aggregate sum of benefits flowing in their direction subject to the group's resource constraint and the agent's incentive compatibility and participation constraints, while taking the other group's contributions as given. This of course presumes that each group is able to develop and establish some kind of cost sharing scheme so as to entice principals of the same kind to enlist in the joint effort. Nevertheless, all this will be considered as a fait accompli and therefore the relevant first-order conditions now become

$$\Phi_1^A - P_1^A - r\sigma_{11}c_{11}P_1^A = 0 \qquad (21)$$

(21a) $$\Phi_2^B - P_2^B - r\sigma_{22}c_{22}P_2^B = 0.$$

It doesn't take much to see that these equations solve to give the second-best solutions for P_1^A and P_2^B.

If the sequence of the proceeding analysis seemed a bit unnatural, it probably should, since we started out first by assuming cooperation among all members and types of principals in order to self-impose certain restrictions on payments made to the agent. Though this improved efficiency in relation to the original third-best outcome, assuming further cooperation in the form of principals of the same type forming their own interest group was still necessary to achieve the second-best outcome. Although the sequence doesn't alter the analytical results – the particular order of the above presentation was chosen to highlight some key points – a more logical development would be the reverse. That is, a more natural progression would entail similar principals forming into groups first and then once these groups are established, inter-group cooperation and bargaining to restrict certain payments made to the agent.[11] This will roughly be the line we take in the next chapter where we explore more explicitly the issue of collective action within the context of common agency.

1.5 Notes

[1] If an agent's actions are in fact complementary, $c_{12} < 0$, then there are economies of scope in having the agent perform multiple tasks or even possibly, in retaining a common agent. As can be seen from the comparative statics in (3a), if an agent's actions are complementary, an increase in the marginal reward for an outcome in one dimension encourages agent effort in the other. The assumption that actions are either independent or substitutes is consistent with assumptions made elsewhere (Dixit (1996) and Holmstrom and Milgrom (1990)). To a certain extent, the interest in assuming a positive cross partial probably stems from the desire to evaluate the worst possible scenario, where tasks may conflict. Although this assumption decreases the desirability of common agency, principals still share the common costs of ensuring the agent's participation and to the extent their interests are similar, the determination of the agent's variable fees.

[2] This can be shown in the following steps for the derivative with respect to P_1^{ki}. First, after taking the derivative, one should obtain:

$$\Phi_1^{ki}\frac{\partial a_1}{\partial P_1}\frac{\partial P_1}{\partial P_1^{ki}} + \Phi_2^{ki}\frac{\partial a_2}{\partial P_1}\frac{\partial P_1}{\partial P_1^{ki}} + P_1^{(-k)}\frac{\partial a_1}{\partial P_1}\frac{\partial P_1}{\partial P_1^{ki}} + P_1^{k(-i)}\frac{\partial a_1}{\partial P_1}\frac{\partial P_1}{\partial P_1^{ki}} + P_2^{(-k)}\frac{\partial a_2}{\partial P_1}\frac{\partial P_1}{\partial P_1^{ki}} + P_2^{k(-i)}\frac{\partial a_2}{\partial P_1}\frac{\partial P_1}{\partial P_1^{ki}} - r\sigma_{11}P_1\frac{\partial P_1}{\partial P_1^{ki}}$$

$$-C_1 \frac{\partial a_1}{\partial P_1}\frac{\partial P_1}{\partial P_1^{ki}} - C_2 \frac{\partial a_2}{\partial P_1}\frac{\partial P_1}{\partial P_1^{ki}} = 0 .$$

Making the appropriate substitutions gives:

$$\Phi_1^{ki}\frac{c_{22}}{\Delta} - \Phi_2^{ki}\frac{c_{12}}{\Delta} + P_1^{(-k)}\frac{c_{22}}{\Delta} + P_1^{k(-i)}\frac{c_{22}}{\Delta} - P_2^{(-k)}\frac{c_{12}}{\Delta} - P_2^{k(-i)}\frac{c_{12}}{\Delta} - r\sigma_{11}P_1 - P_1\frac{c_{22}}{\Delta} + P_2\frac{c_{12}}{\Delta} = 0 .$$

Multiplying through by Δ and grouping like terms gives equation (5) in the text.

[3] It is easier to see these and the results stemming from comparison of solutions (9) and (9a) to those derived under principal collusion (the second-best, sb, case) when using matrix notation. In brief, these are the comparisons of $\mathbf{P}^{fb} = \Phi$ versus $\mathbf{P} = \Phi/(\mathbf{I} + nr\Sigma\mathbf{C})$ and $\mathbf{P}^{sb} = \Phi/(\mathbf{I} + r\Sigma\mathbf{C})$ versus $\mathbf{P} = \Phi/(\mathbf{I} + nr\Sigma\mathbf{C})$ where the **P**'s and Φ are (2x1) matrices with the obvious interpretations; **I** is a (2x2) identity matrix, Σ is a (2x2) diagonal variance-covariance matrix, and **C**, is a (2x2) positive definite matrix of cost terms. In addition, the solution to the agent's problems can be characterized by the equation $\mathbf{a} = \mathbf{C}^{-1}\mathbf{P}$, $\mathbf{a}^{fb} = \mathbf{C}^{-1}\mathbf{P}^{fb}$, $\mathbf{a}^{sb} = \mathbf{C}^{-1}\mathbf{P}^{sb}$. For additional details, one can see Dixit (1996, 1997).

[4] This interpretation assumes that $c_{12} > 0$. If $c_{12} < 0$, these terms would have an opposite reading.

[5] The first-order conditions, (12) – (12c), can be found easily by imposing the above restrictions on equations (5) and (5a) for the appropriate type of principal.

[6] Again an opposite interpretation holds when $c_{12} < 0$.

[7] Whether the agent's variable fees are positive or negative, we will, for the rest of the chapter and through the next, sometimes refer to them without distinction, assuming that in most cases, that context will be clear enough to distinguish between the two. Otherwise payments will refer to positive payments made to the agent, and punishments, as negative payments to the agent or payments made by the agent to principals.

[8] In fact, Bernheim and Whinston demonstrate how common agency can actually tacitly facilitate collusion.

[9] That is, one can envision each principal choosing a function or schedule, $P_i^{\wedge i}(a_1)$, specifying the amount that he is willing to pay for a given level of provision which is also the amount to be provided. Without the problem of asymmetric information each principal's problem can be specified as choosing $P_i^{\wedge i}(a_1)$ and a_1 in order to maximize $\Phi_i^{\wedge i}a_1 + I^{\wedge i} - P_i^{\wedge i}(a_1)$ subject to $\sum_{i \in n^{\wedge}} P_i^{\wedge i} - C(a_1) \geq 0$. After slight manipulation of the first order conditions and the agent's participation constraint, one obtains the desired result: $\sum \Phi_i^{\wedge i} = C_1$. For a more formal treatment, see Dixit, Grossman, and Helpman (1997).

[10] If the standard voluntary contribution model is altered a bit to allow two stages, where in the first, each contributor chooses a matching rate and in the second stage, a flat rate, so that overall a contributor's total payment "is his flat contribution, plus the product of the vector of his matching rates offered to each of his fellows, times the vector of flat contributions (also) chosen by his fellows" (Guttman, 1991, p33), it is not surprising that this game of voluntary provision and matching behavior with identical individuals, results in a Pareto optimal level of provision given the previous discussion that also includes the prior endnote. One can view this matching contribution model as one where implicit or tacit contracting takes place among the contributors without the aid of an agent.

[11] Albeit that at each stage, efficiency improves no matter which sequence is taken, the analysis ignores the possible machinations needed to ensure that voluntary behavior is consistent with improved efficiency. Without these, some parties might actually become worse off and therefore would not enter such agreements unless some of these were imposed say, by a central authority. Given the agent's constraints and the already specified tools needed to align agent incentives and to share the surplus between principals and the agent, the agent will always be held to her reservation utility level. Therefore going from one regime to another, welfare impacts in terms of utility will be felt solely by the principals and these will differ among types. These differences will be highlighted once we suppose that types may act symmetrically (see next chapter). Given the complicated nature of the problem and some expressions, it isn't always possible to definitively compare a given type's utility between different regimes. However with improved efficiency brought about by a change in regime, voluntary behavior will be consistent with our story if the appropriate cash transfers take place among principals of the same type, between types, or between groups of different types. If it is not possible to arrange transfers and the like so that some principals cannot be assured of not being made worse off, our story of voluntary cooperation breaks down. Some other intervention would then have to take place in order to improve on potential third-best

situations. So that the above analyses remain relevant, it must be assumed that voluntary cooperation can at times be possible and if so, that these transfers, although not modeled, take place. We might also suppose that the structure of these transfers and the bargaining that must take place in order to ensure these reforms are successful, will not always be invariant to the order with which cooperation may develop. However we have assumed away any problems that may cause this to be so.

CHAPTER 2: COLLECTIVE ACTION AND COMMON AGENCY, PART 2

2.1 Partial Organization with Simultaneous Moves

According to Olson (1965), smaller groups are better able to overcome the problem of free riding than larger groups. There can be several reasons for this, but a persuasive argument is that with larger groups, individuals will typically view their efforts towards collective provision as having little if any significant impact on the outcome. Yet irrespective of one's behavior, an individual would still anticipate enjoying the public benefits bestowed upon them by the efforts of others. Consequently rational behavior would lead such participants to lower their level of effort or contributions. This would be less likely with smaller groups where it is more probable that individual incentives will be closely aligned with group incentives.

This same logic would also seem to apply within the context of group formation. That is, similar individuals who are less numerous than another segment of the population might find it easier to organize and form an interest group so as to provide stronger incentives to an agent providing them with a localized public good.[1] Similar reasoning would also apply to like-minded principals within a multilateral organization. Using the model developed so far and the previous results, we can examine this proposition. As will be seen in this section, despite an organized group's ability to strengthen the marginal incentives and the agent's action of direct concern to them, they are worse off as a group than if they were acting separately. Moreover, the remaining unorganized group may be better off than they would under standard agency assumptions, that is when all types of principals act separately. Given that the organized group is worse off, we can conclude that there would then be no incentives for these principals to organize unilaterally with respect to the others who remain unorganized. In short, we can expect that under the typical common agency scenario, like that of the standard prisoner's dilemma game, there is no incentive for individuals to unilaterally alter their strategies and cooperate.

We start by first assuming that $n^B > n^A \geq 2$ and that all principals of type A organize and agree to maximize the sum of their aggregate benefits subject to their total resource constraint and the agent's incentive compatibility and participation constraint. Group A also treats the payments made to the agent by unorganized principals as given. So despite the ability of type A principals to organize, the principals still move simultaneously and act as Stackelberg leaders in relation to the agent. Remaining within the confines of our restricted common agency model, we can then represent Group A's problem as choosing Q^A, P_1^A, and P_2^A in order to

$$\text{Maximize} \qquad EU^A = \Phi_1^A a_1 + y^A$$

$$\text{subject to} \qquad Q + P_1 a_1 + P_2 a_2 - (½)\, r[P_1^2\sigma_{11} + P_2^2\sigma_{22}] - C(a_1, a_2) \geq 0$$

$$a_j = a_j(P_j) \;\; j = 1,2$$

plus the group's resource constraint, $Q^A + P_1^A a_1 + P_2^A a_2 + y^A = I^A$. On the other hand, the representative maximization problem for a type B individual is to chose Q^{Bi}, P_1^{Bi}, and P_2^{Bi} to

$$\text{Maximize} \qquad EU^{Bi} = \Phi_2^{Bi} a_2 + y^{Bi}$$

$$\text{subject to} \qquad Q + P_1 a_1 + P_2 a_2 - (½)\, r[P_1^2\sigma_{11} + P_2^2\sigma_{22}] - C(a_1, a_2) \geq 0$$

$$a_j = a_j(P_j) \;\; j = 1,2$$

and the individual's budget constraint, $Q^{Bi} + P_1^{Bi} a_1 + P_2^{Bi} a_2 + y^{Bi} = I^{Bi}$. From these two problems we obtain the following and respective first-order conditions,

(22) $$P_1^A: \quad \Phi_1^A - P_1^A - r\sigma_{11}c_{11}P_1 = 0$$

(22a) $$P_2^A: \quad -P_2^A - r\sigma_{22}c_{22}P_2 = 0$$

(22b) $$P_1^{Bi}: \quad -P_1^{Bi} - r\sigma_{11}c_{11}P_1 = 0$$

(22c) $$P_2^{Bi}: \quad \Phi_2^{Bi} - P_2^{Bi} - r\sigma_{22}c_{22}P_2 = 0.$$

Summing over all type B principals in equations (22b) and (22c) results in

(23) $$-P_1^B - n^B r\sigma_{11}c_{11}P_1 = 0$$

(23a) $$\Phi_2^B - P_2^B - n^B r\sigma_{22}c_{22}P_2 = 0.$$

Then summing over P_1^A and P_1^B and over P_2^A and P_2^B (using expressions (22) and (23) and also (22a) and (23a) respectively) results in the following aggregate variable payments to be made to the agent for a given realization of output under conditions of partial organization, PO, by the principals:

(24) $$P_1^{PO} = \frac{\Phi_1^A}{1+(n^B+1)r\sigma_{11}c_{11}}$$

(24a) $$P_2^{PO} = \frac{\Phi_2^B}{1+(n^B+1)r\sigma_{22}c_{22}}.$$

It is not hard to see when comparing these expressions to the standard third-best outcomes under common agency that $P_1^{PO} > P_1$ and $P_2^{PO} > P_2$. Inasmuch the agent's optimal level of effort is monotonic in P_1 and P_2: $a_1^{PO} > a_1$ and $a_2^{PO} > a_2$. This is a direct result of there being less free-riding or what really amounts to the same thing, less competition in influencing the agent's actions. The reason is that partial organization by the principals lowers the effective cost facing the principals in terms of the agent's risk and cost parameters – there are now (n^A – 1) less principals treating the agency costs as a private cost. Meaning that the n ($n = n^A + n^B$) multiplying these parameters under standard common agency is reduced to ($1 + n^B$) multiplying the same parameters under this partially organized case of common agency, the "1" being interpreted as representing all type A principals acting as a single unit.

If the variable payments of equations (24) and (24a) are broken down into payments made (and received by principals as a group (whether organized or not)), the results for the partially organized case are

(25) $$P_1^{A,PO} = \frac{\Phi_1^A(1 + n^B r\sigma_{11}c_{11})}{1 + (n^B + 1)r\sigma_{11}c_{11}}$$

(25a) $$P_2^{A,PO} = -\frac{\Phi_2^B(r\sigma_{22}c_{22})}{1 + (n^B + 1)r\sigma_{22}c_{22}}$$

(25b) $$P_1^{B,PO} = -\frac{\Phi_1^A(n^B r\sigma_{11}c_{11})}{1 + (n^B + 1)r\sigma_{11}c_{11}}$$

(25c) $$P_2^{B,PO} = \frac{\Phi_2^B(1 + r\sigma_{22}c_{22})}{1 + (n^B + 1)r\sigma_{22}c_{22}} .$$

Comparing these expressions to those obtained under our typical "completely unorganized" common agency case, equations (14)-(14c), we obtain the following results,

$$P_1^A < P_1^{A,PO} \qquad P_2^B > P_2^{B,PO}$$

$$|P_2^A| > |P_2^{A,PO}| \qquad |P_1^B| < |P_1^{B,PO}| .$$

Or in words, type A principals as a group pay higher marginal fees to the agent for g_1 outcomes and receive lower marginal payments from the agent for g_2 outcomes than under standard common agency. The unorganized principals, type B, as a whole, however, receive higher marginal payments for g_1 outcomes and pay lower marginal fees for g_2 outcomes. Although aggregate variable fees are higher in the partially organized case and induce higher levels of effort on the part of the agent, of the higher P_1^{PO}, type B receives some of it by setting a higher punishment for outcomes in the dimension that is not of direct concern to them. The reverse does not hold true for organized type A principals for outcomes of g_2. Even though P_2^{PO} is higher, organized type A principals receive less. These results seem to infer that the increase in efficiency[2] as evidenced by higher aggregate marginal fees and higher levels of agent effort comes at the expense of the organized group and may result in added benefits for those principals that remain unorganized. This may indeed be the case, as we now show.

Using previous results, aggregate utility, by principal type, under standard common agency is

$$U^A = \Phi_1^A a_1 + I^A - P_1^A a_1 - P_2^A a_2 - Q^A$$

$$U^B = \Phi_2^B a_2 + I^B - P_1^B a_1 - P_2^B a_2 - Q^B$$

or

(26) $$U^A = \frac{(\Phi_1^A)^2 n^A r\sigma_{11}}{(1+nr\sigma_{11}c_{11})^2} + \frac{(\Phi_2^B)^2 n^A r\sigma_{22}}{(1+nr\sigma_{22}c_{22})^2} + I^A - Q^A$$

(26a) $$U^B = \frac{(\Phi_2^B)^2 n^B r\sigma_{22}}{(1+nr\sigma_{22}c_{22})^2} + \frac{(\Phi_1^A)^2 n^B r\sigma_{11}}{(1+nr\sigma_{11}c_{11})^2} + I^B - Q^B.$$ [3]

Under the partially organized case, these utilities become

(27) $$U^{A,PO} = \frac{(\Phi_1^A)^2 r\sigma_{11}}{(1+(n^B+1)r\sigma_{11}c_{11})^2} + \frac{(\Phi_2^B)^2 r\sigma_{22}}{(1+(n^B+1)r\sigma_{22}c_{22})^2} + I^A - Q^{A,PO}$$

(27a) $$U^{B,PO} = \frac{(\Phi_2^B)^2 n^B r\sigma_{22}}{(1+(n^B+1)r\sigma_{22}c_{22})^2} + \frac{(\Phi_1^A)^2 n^B r\sigma_{11}}{(1+(n^B+1)r\sigma_{11}c_{11})^2} + I^B - Q^{B,PO}.$$

From these expressions, we prove the first of our major results.

Proposition 1: *Given equal cost sharing[4] of the agent's fixed fee among all principals, type A principals are collectively and individually worse off for having organized than not. Unorganized type B principals on the other hand, are better off, the same as, or worse off when others organize.*

Argument:

Type A principals – to state our case, it is sufficient to argue that

$$U^A - U^{A,PO} = (\Phi_1^A)^2 r\sigma_{11}\left[\frac{n^A}{(1+nr\sigma_{11}c_{11})^2} - \frac{1}{(1+(n^B+1)r\sigma_{11}c_{11})^2}\right]$$

$$+(\Phi_2^B)^2 r\sigma_{22}\left[\frac{n^A}{(1+nr\sigma_{22}c_{22})^2} - \frac{1}{(1+(n^B+1)r\sigma_{22}c_{22})^2}\right]$$

$$-(Q^A - Q^{A,PO}) > 0.$$

It is possible to show that the bracket terms are strictly positive as long as $(n^B)^2 > n^A$ and $n^A > 1$. These are weaker conditions and therefore easily satisfy our prior assumption that $n^B > n^A \geq 2$. Consequently our result hinges on the term $(Q^A - Q^{A,PO})$ being either a small enough positive number, zero, or negative. Given that both Q^A and $Q^{A,PO}$ are indeterminate, we can only surmise that in general these fixed payments are not set arbitrarily but by some kind of negotiating or bargaining process. If it is assumed that the outcome of this process is equal cost sharing[5], then each principal's share of the aggregate fixed payment to the agent is either Q/n or Q^{PO}/n. Thus in order to sign the term $(Q^A - Q^{A,PO})$, we need to sign $(n^A/n)[Q - Q^{PO}]$. Using the agent's participation constraint and the appropriate expression for each level of the agent's action in the appropriate scenario, we obtain

$$(\frac{n^A}{n})[Q - Q^{PO}] = (1/2)(\frac{n^A}{n})\{(r\sigma_{11} - \frac{1}{c_{11}})[(P_1)^2 - (P_1^{PO})^2] + (r\sigma_{22} - \frac{1}{c_{22}})[(P_2)^2 - (P_2^{PO})^2\}.$$

The sign of which – given our results concerning the aggregate variable payments to the agent and our prior assumption of a positive relationship between the variable fees and the fixed fee (see pages 22-23) – is negative. Accordingly then $(Q^A - Q^{A,PO}) < 0$ and therefore $U^A > U^{A,PO}$. Since A as a group suffers overall a loss of utility if organized, identical individual members must also be worse off in the absence of any compensating payments.

Type B Principals – the argument is similar, we need to ascertain the sign of

$$U^B - U^{B,PO} = (\Phi_2^B)^2 n^B r\sigma_{22}\left[\frac{1}{(1+nr\sigma_{22}c_{22})^2} - \frac{1}{(1+(n^B+1)r\sigma_{22}c_{22})^2}\right]$$

$$+ (\Phi_1^A)^2 n^B r\sigma_{11}\left[\frac{1}{(1+nr\sigma_{11}c_{11})^2} - \frac{1}{(1+(n^B+1)r\sigma_{11}c_{11})^2}\right]$$

$$-(Q^B - Q^{B,PO}).$$

Visual inspection of the bracketed terms show that both are negative in sign, therefore we need to determine the sign of $(Q^B - Q^{B,PO})$. Given equal cost sharing and that under partial organization, $Q < Q^{PO}$, we obtain $(Q^B - Q^{B,PO}) < 0$. Whether type B principals are better off in the partially organized case, the same, or worse off, depends on the relative weights between the (negative) terms associated with the agent's variable fee and the (positive) term associated with the principals' payments of the agent's fixed fee. That is, type B principals are better off, the same as, or worse off when type A principals organize then when they do not, depending on whether the negative terms outweigh, are offset by, or are outweighed by the positive term. Depending on outcome of the aggregate utility comparison we can then make the appropriate inferences concerning the welfare of individual type B principals under the two scenarios. ■

With these effects in mind, when at a prior stage where the decision of whether to organize or not must be made before any others, type A principals will perceive themselves as being better off remaining unorganized. Consequently, a smaller group of principals would not opt to organize if they had the choice to do so, even if this induces the agent to increase effort levels in a direction towards being second-best efficient. This result can be seen as perhaps a more insidious form of a failure of collective action. The postulated segment of society that is most likely to organize fails to do so even if the public outcome of an agent's actions benefits them directly and not others.[6] Meaning that even if the good in question is public to certain types but private to others, collective action can still fail. As was shown and argued, this is partly due to the public nature of the fixed cost of achieving agent participation and the inability of the participants to overcome their individual incentives to free ride on the contributions of others irrespective of which type they may be. The other factor contributing to this result is the opportunity that principals have for indirectly extracting income from others by punishing the agent for outcomes that are not of direct concern to them – a "kind of" compensation that principals might demand from the agent, for her serving others even though the agent's tasks are independent of one another. This interpretation, while maybe helpful is slightly misleading. These negative payments simply arise because the principals have the ability to demand them and within the climate examined, it is in their interest to do so. We might also expect or suppose, whether in the presence of an outside central authority or within, say a multilateral agency where it is stated in the charter itself, that such punishment mechanisms are prohibited. If so, a question then arises as to whether the group most likely to organize actually now has the incentive to do so. This is the question we now turn to.

2.1.1 Partial Organization with Simultaneous Moves and Payment Restrictions

It was shown earlier that a restriction of the principals' ability to punish the agent not only partitioned the effects of common agency so as to be limited by type, but it also had the salutary impact of strengthening agent incentives and increasing effort. It then should not be surprising that these effects also occur here if at the outset, there existed an outside agency or central authority capable of making and enforcing this restriction. Assuming this is so, then in the simultaneous move game, the restriction $P_1^{Bi} = 0$ is set for each $i \in n^B$ and for the organized principals, the restriction, $P_2^A = 0$, holds. The resulting first-order conditions for both types are then

(28) $$P_1^A: \quad \Phi_1^A - P_1^A - r\sigma_{11}c_{11}P_1^A = 0$$

(28a) $$P_2^{Bi}: \quad \Phi_2^{Bi} - P_2^{Bi} - r\sigma_{22}c_{22}P_2^B = 0.$$

Summing over all n^B in equation (28a) and solving each equation separately to find the agent's unit fees give

(29) $$P_1^{POR} = P_1^{A,POR} = \frac{\Phi_1^A}{1 + r\sigma_{11}c_{11}}$$

(29a) $$P_2^{POR} = P_2^{B,POR} = \frac{\Phi_2^B}{1 + n^B r\sigma_{22}c_{22}}$$

where the "R" added to the superscript again refers to the fact that certain payments are restricted. These results are an improvement when compared to the unrestricted common agency case. However, if our benchmark case were the restricted but unorganized common agency case discussed earlier, the solutions denoted by "RU" – equations (19) and (19a) – we see that the only change is a strengthening of the unit payment for g_1 outcomes and therefore: $a_1^{POR} > a_1^{RU}$ and $a_2^{POR} = a_2^{RU}$. Using this case as our benchmark, we can state the following result for the simultaneous move game played between organized and unorganized principals.

Proposition 2: *If $1 < n^A(r\sigma_{11}c_{11})^2$, then organizing type A principals are either better off, the same, or worse off than would be the case if punishments were restricted and both types remained unorganized and moved simultaneously. Unorganized type B principals are worse off.*

Argument: The argument for comparison of type B principals rests on the determination of $-\text{sign}(Q^{B,RU} - Q^{B,POR})$. Retaining the equal cost sharing assumption gives the following result for both types of principals (k = A, B):

$$Q^{k,RU} - Q^{k,POR} = (1/2)(\frac{n^k}{n})[Q^{RU} - Q^{POR}]$$

$$= (1/2)(\frac{n^k}{n})(r\sigma_{11} - \frac{1}{c_{11}})[(P_1^{RU})^2 - (P_1^{POR})^2] < 0.$$

Thus for type B principals,

$$U^{B,RU} - U^{B,POR} = -(Q^{B,RU} - Q^{B,POR}) > 0.$$ [7]

Given the assumption, $1 < n^A(r\sigma_{11}c_{11})^2$, and the arguments made above regarding the agent's fixed payments, it then follows that the expression

$$U^{A.RU} - U^{A.POR} = (\Phi_1^A)^A r\sigma_{11}\left[\frac{n^A}{(1+n^A r\sigma_{11}c_{11})^2} - \frac{1}{(1+r\sigma_{11}c_{11})^2}\right]$$

$$-(Q^{A.RU} - Q^{A.POR})$$

is negative, zero, or positive depending on whether the negative first term dominates, is offset, or outweighed by the positive second term. ■

On an additional note, it is possible to give alternative interpretations for the requirement, $1 < n^A(r\sigma_{11}c_{11})^2$. The first is, that this holds as long as n^A is sufficiently large, that the restricted but unorganized common agency, has a significant amount of principals vying for the agent's attentions. This would be required if it were the case that $0 < r\sigma_{11}c_{11} < 1$. However, if $r\sigma_{11}c_{11} > 1$, this implies that $\partial Q/\partial P_1 > 0$, which we earlier argued was the more intuitive case – that the impact on agent's risk premium is greater than the response of the agent to an additional unit of P_1.

Except for the possibility where the negative term associated with the agent's variable fees does not dominate the increase in the fixed fee payments, the last proposition shows that when punishments are restricted, collective action is likely to improve matters. Thus in a simultaneous move game and using the analogy of working within the context of a multilateral agency, we can say that with restrictions, the organized group's interests can be made to be roughly consistent with the organization's. A similar interpretation would also hold for organized groups in a community setting with local public goods.

2.2 Intervention by a Central Authority

The first section of this chapter contains a rather gloomy prognosis for collective action under the conditions given, especially in the absence of any restriction on certain marginal payments, i.e. sufficient incentives fail to exist for the most likely types to coalesce and improve upon the third-best allocation. Although this does not prove the case for central intervention (since cash transfers among principals can still conceivably take place), it is natural to inquire or at least consider a particular form (or forms) that intervention may take. In this section, one main alternative is considered, so remedies are by no means unique, but what may initially recommend this one, as opposed to some other form, is that it attempts to implement a solution that is consistent with decentralized behavior. It is claimed that this is an improvement over the typical "black-box" method used earlier, from which we were able to obtain our earlier second-best benchmarks. This was accomplished through the use of the simplifying assumption of unified principal behavior, but one could very well have also assumed the presence of a benevolent social planner. In this section, such behavior or acquiescence by the principals to the planner's wishes are not taken for granted. More importantly as long as the central authority can restrict payments that are "punishing" rather than "rewarding", it doesn't matter if one group organizes or not - the model to be presented is still capable of implementing second-best levels of agent effort. At least with respect to incentives that affect the actions of the agent at the margin, the restrictions effectively destroy the linkage between the two types of principals via the agent. However, there remains the question of how the burden of the agent's fixed fee is to be distributed across principals. Yet again this aspect of the problem requires additional structure so that this determination can be made.

That said, given the four-stage game to be later described and analyzed, it is possible to attain the second-best outcome. The central authority moves twice: first when setting the restrictions on punishments, and subsequently in the third stage, after the principals, when choosing the proper level of subsidies to make to the agent. In the second stage principals must correctly anticipate the planner's response to their choices. Both nonetheless, must condition their payments to the agent on observable output rather than on the agent's (nonobservable) effort. The game however, has multiple equilibria. Yet even here, because of the sequential movement between principals and the central planner and the partitioning effect of the restrictions, this is not quite the problem it can be in a simultaneous move game played amongst players (principals) of many different types.

But before we turn to describing the overall structure of the model, we first investigate another approach for strengthening agent incentives which also allows for decentralized behavior. This approach involves direct payment of subsidies to principals and is augmented by suitable payment restrictions. With these two tools, subsidies and restrictions, it can be easily shown that it is possible to entice the agent into choosing levels of effort that are second best. There is however one drawback in implementing this scheme: the level of subsidies required to induce the principals into increasing their marginal rewards may, in the final analysis, be too high in relation to the overall value that the principals place on a unit of the agent's output. Far from setting up a straw man, the important lesson to be learned is that in evaluating the effectiveness and worthiness of implementing such a subsidy, this consideration should not be ignored.

2.2.1 Common Agency and Piquovian Subsidies to Principals

Recall that an imposition of the restrictions: $P_2^{Ai} = 0$ for every $i \in n^A$ and $P_1^{Bi} = 0$ for all $i \in n^B$, is not sufficient for achieving a second-best outcome when there is more than one principal of each type. Therefore in order to reach the second-best allocation, this policy tool is supplemented with an additional one. Given that the principals of a certain type still pay too little an amount when rewarding the agent's effort in the dimension pertinent to them, assume that the central authority offers each principal of a given type a unit subsidy. That is, for each unit of the agent's output, the central authority pledges to pay a subsidy of s_1^{Ai} to a principal belonging to type A, and a subsidy s_2^{Bi} to a principal who is type B. These subsidies effectively reduce the cost of rewarding agent effort for each principal and therefore encourages them to spend more.

Other than the addition of these subsidies and restrictions, the principals solve a similar problem as before. For example, a principal of type A chooses Q^{Ai} and P_1^{Ai} in order to

$$\text{Maximize} \qquad \Phi_1^{Ai} a_1 + I^{Ai} - (P_1^{Ai} - s_1^{Ai}) a_1 - Q^{Ai}$$

$$\text{s.t} \;\; Q + P_1^A a_1 + P_2^B a_2 - (1/2) r[\sigma_{11}(P_1^A)^2 + \sigma_{22}(P_2^B)^2] - C(a_1, a_2) \geq 0$$

$$a_1 = a_1(P_1^A).$$

Observe that the principal's budget constraint has been substituted into the objective function and that the effect of the planner's restrictions have already been incorporated into the agent's incentive compatibility constraint along with the assumption that $c_{12} = 0$. Following already established procedures, we then can write the first-order conditions for both types of principals:

(30) $$\Phi_1^{Ai} + s_1^{Ai} - P_1^{Ai} - r\sigma_{11} c_{11} P_1^A = 0$$

(30a) $$\Phi_2^{Bi} + s_2^{Bi} - P_2^{Bi} - r\sigma_{22} c_{22} P_2^B = 0.$$

Thus if the central planner pays a unit subsidy of $s_1^{Ai} = r\sigma_{11} c_{11} P_1^{A(-i)}$ and $s_2^{Bi} = r\sigma_{22} c_{22} P_2^{B(-i)}$ to each principal of type A and B respectively[8], it is possible to attain the second-best variable fees and effort levels of the agent:

$$P_1^{Ai,sb} = \frac{\Phi_1^{Ai}}{1 + r\sigma_{11}c_{11}} \quad \Rightarrow \quad P_1^{A,sb} = \frac{\Phi_1^{A}}{1 + r\sigma_{11}c_{11}}, \quad a_1^{sb} = (1/c_{11})P_1^{A,sb}$$

$$P_2^{Bi,sb} = \frac{\Phi_2^{Bi}}{1 + r\sigma_{22}c_{22}} \quad \Rightarrow \quad P_2^{B,sb} = \frac{\Phi_2^{B}}{1 + r\sigma_{22}c_{22}}, \quad a_2^{sb} = (1/c_{22})P_2^{B,sb}.$$

Comparing $P_1^{Ai,sb}$ to P_1^{Ai} under normal common agency ($P_1^{Ai} = \frac{\Phi_1^{Ai}}{1 + nr\sigma_{11}c_{11}}$) or to where payments are already restricted, $P_1^{Ai,RU}$ ($P_1^{Ai,RU} = \frac{\Phi_1^{Ai}}{1 + n^A r\sigma_{11}c_{11}}$), it is easy to see that the subsidy induces each principal to strengthen their marginal reward to the agent. However, in order to realize this proper alignment of incentives requires the central authority to expend s_1 and s_2 per unit of the agent's expected output where $s_1 = \sum_{i \in n^A} s_1^{Ai} = (n^A - 1)r\sigma_{11}c_{11}P_1^A$ and $s_2 = \sum_{i \in n^B} s_2^{Bi} = (n^B - 1)r\sigma_{22}c_{22}P_2^B$. As can be seen, if $r\sigma_{11}c_{11} > 1$, the governmental authority can expect to pay a substantial multiple of the aggregate variable fees paid by each type of principal. In addition, these subsidies exceed the aggregate value that the principals place on a unit of output (Φ_j^k, k = A, B, j = 1, 2 but when k = A, j ≠ 2 and when k = B, j ≠ 1).

Moreover if these subsidies must be financed by taxes on these same principals, whether they are distortionary or not, it is clear that this policy would no longer be practical let alone politically feasible. Hence in the absence or falling short of finding and taxing fixed rents from another sector in the economy (in which case the policy may be justified if the transfer can be made in a lump sum fashion), there is not much to recommend this modified subsidy policy for increasing the agent's levels of effort. Consequently we turn towards describing an alternative approach, and though this approach includes the possibility of free riding by the principals, the onus of funding subsidies will not be as great.

2.2.2 Subsidizing the Common Agent

Consider that instead of paying subsidies to each principal, the central authority pays a unit subsidy for each dimension of output directly to the agent and that payment is made only after the principals have already pledged their contributions. The structure of the game is described as follows:

Stage 1: The central authority moves first, its goal is to implement a second-best equilibrium. In order to do so, it restricts payments to the agent that are not directly and positively linked to output and a given type's well-being, i.e. $P_2^{Ai} = 0$ for every $i \in n^A$ and $P_1^{Bi} = 0$ for all $i \in n^B$.

Stage 2: Each principal chooses Q^{ki} and P_j^{ki} ($k = A, B$, $j = 1, 2$ but when $k = A$, $j \neq 2$ and when $k = B$, $j \neq 1$) in order to maximize individual utility while anticipating the responses of both the agent and the central authority and taking as given the government's prior restrictions and payments by other principals.

Stage 3: Given payments by the principals, the central authority makes additional variable payments, s_1 and s_2, to the agent. These are chosen to maximize the aggregate utility of the principals subject to the following constraints: the central authority's budget constraints, the principals' resource constraint, and the agent's participation and incentive compatibility constraints.

Stage 4: Agent chooses a_1 and a_2 given the choices made by all principals and the central authority.

Again the solution concept is sub-game perfection, we therefore start by characterizing the agent's behavior under the impact of receiving an additional variable fee from another party. The agent's certainty equivalent of wealth is now written as

$$u^{CEW} = Q + (P_1^A + s_1)a_1 + (P_2^B + s_2)a_2 - (1/2)r[\sigma_{11}(P_1^A + s_1)^2 + \sigma_{22}(P_2^B + s_2)^2] - C(a_1, a_2).$$

From which, we derive the first-order conditions:

$$(P_1^A + s_1) - C_1(a_1, a_2) = 0 \quad (31)$$

$$(P_2^B + s_2) - C_2(a_1, a_2) = 0. \quad (31a)$$

As can be seen, the agent's optimal responses to changes in the central authority's policy variables are equivalent to responses towards changes in the relevant aggregate payments chosen by the principals,

$$\partial a_1/\partial s_1 = \partial a_1/\partial P_1^A = 1/c_{11} \quad (32)$$

$$\partial a_2/\partial s_2 = \partial a_2/\partial P_2^B = 1/c_{22}. \quad (32a)$$

We can also see from the specification of the problem and the above conditions that the agent is indifferent as to the source from which she receives these funds.

At the third stage of the game the central authority's problem is to choose s_1 and s_2 to

$$\text{Maximize} \quad \Phi_1^A a_1 + \Phi_2^B a_2 + I^A + I^B - Q^A - Q^B - P_1^A a_1 - P_2^B a_2 - t_1^A a_1 - t_2^B a_2$$

$$\text{s.t. } Q + (P_1^A + s_1)a_1 + (P_2^B + s_2)a_2 - (1/2)r[\sigma_{11}(P_1^A + s_1)^2 + \sigma_{22}(P_2^B + s_2)^2] - C(a_1, a_2) \geq 0$$

$$a_1 = a_1(P_1^A, s_1),\ a_2 = a_2(P_2^B, s_2),\ t_1^A = s_1,\ t_2^B = s_2.$$

Note that the central authority's budget constraints, $t_1^A = s_1$ and $t_2^B = s_2$, not only ensure that its variable fees to the agent are fully funded, but also that no group is favored at another's expense. After incorporation of all constraints into the objective function and taking derivatives with respect to the two choice variables, the resulting first-order conditions are

$$s_1: \quad \Phi_1^A \frac{\partial a_1}{\partial s_1} - r\sigma_{11}(P_1^A + s_1) - C_1 \frac{\partial a_1}{\partial s_1} = 0$$

$$s_2: \qquad \Phi_2^B \frac{\partial a_2}{\partial s_2} - r\sigma_{22}(P_2^B + s_2) - C_2 \frac{\partial a_2}{\partial s_2} = 0.$$

Using results from equations (31) through (32a) these conditions can be rewritten as

(33) $$s_1: \qquad \Phi_1^A - r\sigma_{11}c_{11}(P_1^A + s_1) - (P_1^A + s_1) = 0$$

(33a) $$s_2: \qquad \Phi_2^B - r\sigma_{22}c_{22}(P_2^B + s_2) - (P_2^B + s_2) = 0.$$

Solving each of these for s_1 and s_2 respectively, gives the central authority's response functions to changes in P_1^A and P_2^B:

(34) $$s_1 = \frac{\Phi_1^A - (1 + r\sigma_{11}c_{11})P_1^A}{1 + r\sigma_{11}c_{11}}$$

(34a) $$s_2 = \frac{\Phi_2^B - (1 + r\sigma_{22}c_{22})P_2^B}{1 + r\sigma_{22}c_{22}},$$

with partial derivatives of

(35) $$\frac{\partial s_1}{\partial P_1^A} = \frac{\partial s_2}{\partial P_2^B} = -1.$$

The expressions for (35) show that subsidies to the agent respond negatively, dollar-for-dollar, to changes in P_1^A and P_2^B. This "crowding out" effect however, is just a result of the agent's indifference to the sources of her funding.

Implicit within the context of the solution to the central planner's problem is the setting of the individual tax schedules for the principals: t_1^{Ai} and t_2^{Bi}. Insofar as s_1 is a function of P_1^A and s_2 is a function of P_2^B and given the two budget constraints, we can specify $t_1^{Ai}(P_1^{Ai})$ and $t_2^{Bi}(P_2^{Bi})$ as being representations of the schedules facing each individual principal as they solve their individual maximization problem. Each principal, depending on type, will take into account the appropriate responses of the authority when determining their variable payments in the second stage. Although s_1 and s_2 may decline for every unit that P_1^{Ai} and P_2^{Bi} are increased, t_1^{Ai} and t_2^{Bi} will also be respectively a function of these payments and so one way or another, marginal incentives for the agent will be provided for.

Without loss of generality we focus on the problem facing a type A principal. The budget constraint for such an individual is now written as $y^{Ai} + Q^{Ai} + P_1^{Ai}a_1 - t_1^{Ai}a_1 = I^{Ai}$. After substituting Ai's budget constraint and the agent's participation constraint into Ai's utility function, the principal's problem is then to choose P_1^{Ai} to

$$\text{Maximize} \quad \Phi_1^{Ai}a_1 + s_1a_1 - t_1^{Ai}a_1 + P_1^{A(-i)}a_1 + Q^{A(-i)} + Q^B + (P_2^B + s_2)a_2 + I^{Ai}$$

$$-(1/2)r[\sigma_{11}(P_1^A + s_1)^2 + \sigma_{22}(P_2^B + s_2)^2] - C(a_1, a_2)$$

$$\text{s.t.} \quad a_1 = a_1(P_1^A, s_1), \;\; a_2 = a_2(P_2^B, s_2), \;\; t_1^{Ai} = t_1^{Ai}(P_1^{Ai})$$

$$0 \le P_1^{Ai} \le \Phi_1^{Ai}.$$

The first-order conditions that characterize the solution for a typical principal of type A at this stage of the game are

$$(36) \quad \Phi_1^{Ai}\left(\frac{\partial a_1}{\partial P_1^A} + \frac{\partial a_1}{\partial s_1}\frac{\partial s_1}{\partial P_1^A}\right) + s_1\left(\frac{\partial a_1}{\partial P_1^A} + \frac{\partial a_1}{\partial s_1}\frac{\partial s_1}{\partial P_1^A}\right) - t_1^{Ai}\left(\frac{\partial a_1}{\partial P_1^A} + \frac{\partial a_1}{\partial s_1}\frac{\partial s_1}{\partial P_1^A}\right) + \left(\frac{\partial s_1}{\partial P_1^A} - \frac{\partial t_1^{Ai}}{\partial P_1^{Ai}}\right)a_1$$

$$- r\sigma_{11}(P_1^A + s_1)\left(1 + \frac{\partial s_1}{\partial P_1^A}\right) - C_1\left(\frac{\partial a_1}{\partial P_1^A} + \frac{\partial a_1}{\partial s_1}\frac{\partial s_1}{\partial P_1^A}\right) + \lambda_1^{Ai} - \mu_1^{Ai} = 0$$

$$(36a) \qquad \lambda_1^{Ai}P_1^{Ai} = 0 \qquad \lambda_1^{Ai} \ge 0 \qquad P_1^{Ai} \ge 0$$

$$(36b) \qquad \mu_1^{Ai}(\Phi_1^{Ai} - P_1^{Ai}) = 0 \qquad \mu_1^{Ai} \ge 0 \qquad \Phi_1^{Ai} - P_1^{Ai} \ge 0.$$

The Lagrange multipliers λ_1^{Ai} and μ_1^{Ai}, are associated with the lower and upper bound restrictions for P_1^{Ai}. Given that $\frac{\partial a_1}{\partial P_1^A} = \frac{\partial a_1}{\partial s_1}$ and $\frac{\partial s_1}{\partial P_1^A} = -1$, the first-order conditions can be rewritten as

(37) $$\left(\frac{\partial s_1}{\partial P_1^A} - \frac{\partial t_1^{Ai}}{\partial P_1^{Ai}}\right) + \lambda_1^{Ai} - \mu_1^{Ai} = 0$$

(37a) $$\lambda_1^{Ai} P_1^{Ai} = 0 \qquad \lambda_1^{Ai} \geq 0 \qquad P_1^{Ai} \geq 0$$

(37b) $$\mu_1^{Ai}(\Phi_1^{Ai} - P_1^{Ai}) = 0 \qquad \mu_1^{Ai} \geq 0 \qquad \Phi_1^{Ai} - P_1^{Ai} \geq 0 .$$

If the central authority's policy responses to a change in Ai's variable payment are offsetting, then the first term in equation (37) is equal to zero. It is assumed that this is the case – that the authority increases the amount of the subsidy at the same rate it increase Ai's taxes for every unit not paid voluntarily by the principal towards the marginal reward of the agent. Thus $\lambda_1^{Ai} = \mu_1^{Ai}$, and from the remaining conditions we secure the result: $P_1^{Ai} \in [0, \Phi_1^{Ai}]$. Similar arguments also give $P_2^{Bi} \in [0, \Phi_2^{Bi}]$. These hold for all principals according to their type.

At first glance, the continua of multiple equilibria are somewhat disconcerting. Each of these is however limited to a certain number of principals of a given type. This should facilitate the choice of which equilibrium to focus on, given similar interests. Further, given that these choices will be confined to being made noncooperatively among principals of the same type, it is proposed that of these, three equilibria emerge as likely candidates. Focusing only on choices of type A principals, they, along with their following implications for s_1, P_1^A, and $s_1 + P_1^A$, are:

i) if $P_1^{Ai} = 0 \;\forall\, i \in n^A$,

$$\text{then} \quad s_1 = \frac{\Phi_1^A}{1 + r\sigma_{11}c_{11}}, \quad P_1^{Ai} = 0, \quad \text{and } s_1 + P_1^A = \frac{\Phi_1^A}{1 + r\sigma_{11}c_{11}}$$

ii) if $P_1^{Ai} = \Phi_1^{Ai} \ \forall \ i \in n^A$,

$$\text{then} \quad s_1 = -\left(\frac{r\sigma_{11}c_{11}}{1 + r\sigma_{11}c_{11}}\right)\Phi_1^A, \quad P_1^A = \Phi_1^A, \quad \text{and } s_1 + P_1^A = \frac{\Phi_1^A}{1 + r\sigma_{11}c_{11}}$$

iii) if $P_1^{Ai} = \dfrac{\Phi_1^{Ai}}{1 + r\sigma_{11}c_{11}} \ \forall \ i \in n^A$,

$$\text{then} \quad s_1 = 0, \quad P_1^A = \frac{\Phi_1^A}{1 + r\sigma_{11}c_{11}} \quad \text{and } s_1 + P_1^A = \frac{\Phi_1^A}{1 + r\sigma_{11}c_{11}}.$$

Each of these cases is described separately. In case (i), each principal chooses not to pay the agent directly. The central government takes up the slack and taxes each principal accordingly. In this case, the problem reduces to a typical social planning problem with fully funded subsidies and no interaction between the principals and the agent. For case (ii), Ai pays a variable fee equal to its marginal valuation of expected output. In effect, each principal, in setting $P_1^{Ai} = \Phi_1^{Ai}$, also relinquishes "individual control" in designing the agent's incentives knowing that even though it "overpays" in the sense of the structure and overall goal of the game, the central authority will tax the agent and rebate the amount overpaid back to the principal. For scenario (iii), each principal chooses a level of P_1^{Ai} that is consistent with implementing the second-best outcome and accordingly no intervention is required by the central authority. Thus assuming that all principals of a certain type focus on a single equilibrium and that each can also agree, whether cooperatively or noncooperatively, on the allocation of payments for the agent's fixed fee, it is possible to implement the second-best allocation.

The central planner in setting the rules or the institutional framework for which the game between itself, the principals, and the agent is to be played, chooses the restrictions $P_2^{Ai} = P_1^{Bi} = 0$ in order to implement the second-best allocation. As has been shown, these choices are consistent with the planner's objective.

With the administration of restrictions, subsidies, and taxes, the central authority may appear to have a lot on its hands. A less sanguine view may be more realistic. For example, a more practical alternative is for the authority to limit itself to restricting certain payments and encouraging collective action among principals of each type. If successful, even the larger group may overcome the free riding problem and second-best efficiency may be achieved without the use of subsidies and taxes, at least as these tools are described here.

Finally, we have yet to discuss the determination of the allocation of the agent's fixed fee to be paid by the principals. As argued earlier, this can be settled by negotiation or bargaining among the parties. In the case of the central authority ensuring the strength of the marginal incentives, the problem then becomes a cost allocation game between the principals. The outcome determining the distribution of the fixed fee, again, presumably depends on the relative bargaining powers of the principals and is possibly influenced by whether certain types of principals organize or not, and if so, whether it gives them a strategic advantage over principals that remain unorganized. If the central authority only provides restrictions, we are then back to the case summarized in the Proposition 2 at the end of the last section.

2.3 Partial Organization with Sequential Moves

In Section 2.1, it was assumed that one of two types of principals could choose to organize in a prior stage and in the subsequent stage, both types played a simultaneous game

– choosing payments to be made to the agent while taking the others' payments as given while fully anticipating the effects that such payments have on agent effort and participation. In this section, we follow a similar line of inquiry concerning the viability of partial cooperation albeit in a game where the two types of principals move sequentially. Within this context, we retain the assumption that the smaller numbers of type A principals enable them to be the most likely candidates to organize. In addition, it is also assumed that they are the most inclined to move first. Although the choice of roles is not modeled here, we might posit that type A principals fulfill an incumbency-like position with regard to setting up and running the common agency that provides benefits to other potential members. In such a scenario, type A principals when moving first, have the incentive to consider the impact of their payments on the agent and the other principals in addition to ensuring the participation of both the agent and type B principals.

As usual, the equilibrium concept used is sub-game perfection and since the agent's behavior has already been characterized in Section 2.1, we now proceed with the characterization of behavior of unorganized type B principals since they move after organized type A principals but before the agent. Given that every member i of type B principals all move after group A, each then essentially faces a similar problem as they did in the Section 2.1. Thus each Bi chooses Q^{Bi}, P_1^{Bi}, and P_2^{Bi} subject to the agent's participation and incentive compatibility constraints and their own budget constraint while taking the organized group's choices as given. Following similar procedures as before, this gives what should be by now the familiar first-order conditions for type B principals,

(38) $$P_1^{Bi}: \quad -P_1^{Bi} - r\sigma_{11}c_{11}P_1 = 0$$

(38a) $$P_2^{Bi}: \quad \Phi_2^{Bi} - P_2^{Bi} - r\sigma_{22}c_{22}P_2 = 0.$$

Summing over all Bi, we then can obtain expressions for the aggregate variable payments paid by type B principals as a function of group A's variable payments. These are:

(39) $$P_1^B = -\frac{n^B r\sigma_{11}c_{11}P_1^A}{1+n^B r\sigma_{11}c_{11}}$$

(39a) $$P_2^B = \frac{\Phi_2^B - n^B r\sigma_{22}c_{22}P_2^A}{1+n^B r\sigma_{22}c_{22}}.$$

From these expressions, we can also obtain the optimal aggregate response of type B principals to changes in type A's payments,

(40) $$\frac{\partial P_1^B}{\partial P_1^A} = -\frac{n^B r\sigma_{11}c_{11}}{1+n^B r\sigma_{11}c_{11}}$$

(40a) $$\frac{\partial P_2^B}{\partial P_2^A} = -\frac{n^B r\sigma_{22}c_{22}}{1+n^B r\sigma_{22}c_{22}}.$$

The slope of these response functions state that unorganized type B principals as a whole respond negatively to changes in group A's marginal incentive payments to the agent.

As in the simultaneous move game, the allocation of the agent's fixed fee remains undetermined among unorganized type B principals. Since the burden will be shared amongst principals of the same type, the most likely candidate for selection is the rule of equal cost sharing. Assuming that group A's fixed fee is determined in the first stage along with P_1^A and P_2^A – their marginal payments to the agent – and given the fact that type B principals determine the allocation of their portion of the fixed fee such that the agent's participation constraint is binding, we can write

(41) $$Q^B = -Q^A - P_1^B a_1 - P_1^A a_1 - P_2^B a_2 - P_2^A a_2 + (1/2)r[\sigma_{11}P_1^2 + \sigma_{22}P_2^2] + C(a_1, a_2),$$

the expression for that portion of the agent's fixed fee paid by type B principals as a function of group A's marginal and fixed payments to the agent.

Group A thus takes into account the overall response of type B principals to its choice of P_1^A, P_2^A, and Q^A, and that whatever the choices, if B participates, type B principals will ensure the agent's participation. So that group A does not set Q^A to negative infinity, a realistic lower bound must be established. One way to realistically represent this, is to restrict A's choice set (P_1^A, P_2^A, Q^A) so that it also ensures the participation of type B principals given $P_1^B(P_1^A)$, $P_2^B(P_2^A)$, and the agent's incentive compatibility constraint. Group A's problem then is to

$$\text{Maximize} \quad \Phi_1^A a_1 + I^A - P_1^A a_1 - P_2^A a_2 - Q^A$$

$$\text{subject to} \quad \Phi_2^B a_2 + I^B - P_1^B a_1 - P_2^B a_2 - Q^B \geq 0$$

$$a_j = a_j(P_j) \text{ and } P_j^B = P_j^B(P_j^A) \;\; j = 1,2$$

and equation (41).[9]

Using (41) to substitute out for Q^B in the aggregate indirect utility function of type B principals, the problem can be rewritten as

$$\text{Maximize} \quad \Phi_1^A a_1 + I^A - P_1^A a_1 - P_2^A a_2 - Q^A$$

$$\text{subject to} \quad \Phi_2^B a_2 + I^B + Q^A + P_1^A a_1 + P_2^A a_2 - (1/2)r[\sigma_{11}P_1^2 + \sigma_{22}P_2^2] - C(a_1, a_2) \geq 0$$

$$a_j = a_j(P_j) \text{ and } P_j^B = P_j^B(P_j^A) \;\; j = 1,2.$$

Using the participation constraint[10] for type B principals to solve for Q^A and plugging the result into group A's objective function, the problem now becomes

$$\text{Maximize} \quad \Phi_1^A a_1 + \Phi_2^B a_2 + I^A + I^B - (1/2)r[\sigma_{11}P_1^2 + \sigma_{22}P_2^2] - C(a_1, a_2)$$

$$\text{subject to} \qquad a_j = a_j(P_j) \text{ and } P_j^B = P_j^B(P_j^A)\ j = 1,2\,.$$

Given the above structure of the sequential move game, it is now easy to see that it collapses into a problem where the outcome will be constrained Pareto efficient – i.e. group A's choices will implement the second-best outcome. This is, in fact, confirmed, as we now show after deriving the initial first-order conditions:

$$P_1^A: \quad \Phi_1^A\left(\frac{\partial a_1}{\partial P_1}\frac{\partial P_1}{\partial P_1^A}+\frac{\partial a_1}{\partial P_1}\frac{\partial P_1}{\partial P_1^B}\frac{\partial P_1^B}{\partial P_1^A}\right)-r\sigma_{11}P_1\left(\frac{\partial P_1}{\partial P_1^A}+\frac{\partial P_1}{\partial P_1^B}\frac{\partial P_1^B}{\partial P_1^A}\right)$$

$$-C_1\left(\frac{\partial a_1}{\partial P_1}\frac{\partial P_1}{\partial P_1^A}+\frac{\partial a_1}{\partial P_1}\frac{\partial P_1}{\partial P_1^B}\frac{\partial P_1^B}{\partial P_1^A}\right)=0$$

$$P_2^A: \quad \Phi_2^B\left(\frac{\partial a_2}{\partial P_2}\frac{\partial P_2}{\partial P_2^A}+\frac{\partial a_2}{\partial P_2}\frac{\partial P_2}{\partial P_2^B}\frac{\partial P_2^B}{\partial P_2^A}\right)-r\sigma_{22}P_2\left(\frac{\partial P_2}{\partial P_2^A}+\frac{\partial P_2}{\partial P_2^B}\frac{\partial P_2^B}{\partial P_2^A}\right)$$

$$-C_2\left(\frac{\partial a_2}{\partial P_2}\frac{\partial P_2}{\partial P_2^A}+\frac{\partial a_2}{\partial P_2}\frac{\partial P_2}{\partial P_2^B}\frac{\partial P_2^B}{\partial P_2^A}\right)=0\ .$$

Using the expressions from the agent's first-order conditions ($\partial a_j/\partial P_j = 1/c_{jj}$ and $P_j = C_j$ where $j = 1, 2$) and rearranging, we obtain

(42) $$\Phi_1^A - r\sigma_{11}c_{11}P_1 - P_1 = 0$$

(42a) $$\Phi_2^B - r\sigma_{22}c_{22}P_2 - P_2 = 0\,.$$

Solving each for P_j, $j = 1, 2$ we derive the expressions for the aggregate marginal payments to the agent:

(43) $$P_1^{POS} = \frac{\Phi_1^A}{1+r\sigma_{11}c_{11}}$$

(43a) $$P_2^{POS} = \frac{\Phi_2^B}{1+r\sigma_{22}c_{22}}$$

where the superscript "POS" denotes the solution pertaining to the partially organized case with sequential moves. Using the relationships

$$P_1 = P_1^A + P_1^B(P_1^A)$$

$$P_2 = P_2^A + P_2^B(P_2^B)$$

or

$$P_1^A = P_1 - P_1^B(P_1^A)$$

$$P_2^A = P_2 - P_2^B(P_2^A)$$

and (39) and (39a) we find that

(44) $$P_1^{A,POS} = \frac{\Phi_1^A(1+n^B r\sigma_{11}c_{11})}{1+r\sigma_{11}c_{11}}$$

(44a) $$P_2^{A,POS} = \frac{\Phi_2^B(n^B-1)r\sigma_{22}c_{22}}{1+r\sigma_{22}c_{22}}.$$

And by using (44) and (44a) respectively in (39) and (39a), we also get

(45) $$P_1^{B,POS} = -\frac{\Phi_1^A n^B r\sigma_{11}c_{11}}{1+r\sigma_{11}c_{11}}$$

(45a) $$P_2^{B,POS} = \frac{\Phi_2^B}{1+n^B r\sigma_{11}c_{11}}\left[\frac{1+r\sigma_{22}c_{22} - n^B(n^B-1)(r\sigma_{22}c_{22})^2}{1+r\sigma_{22}c_{22}}\right].$$

Finally, using the appropriate expressions we determine the expressions for the agent's actions,

(46) $$a_1^{POS} = \frac{1}{c_{11}}P_1^{POS}$$

(46a) $$a_2^{POS} = \frac{1}{c_{22}}P_2^{POS}$$

and after substituting out for each a_j^{POS}, j = 1, 2, and after some slight rearranging, we also obtain the expression for group A's portion of the agent's fixed fee:

$$(47)\qquad Q^{A,POS} = -\Phi_2^B \frac{1}{c_{11}} P_1^{POS} - I^B - P_1^{A,POS} \frac{1}{c_{11}} P_1^{POS} - P_2^{A,POS} \frac{1}{c_{22}} P_2^{POS}$$

$$+ (1/2)(P_1^{POS})^2[r\sigma_{11} + \frac{1}{c_{11}}] + (1/2)(P_2^{POS})^2[r\sigma_{11} + \frac{1}{c_{22}}].$$

Contrary to what one may expect, we see by observing (45) and (45a), that not only do type B principals receive a "subsidy" or a payment from the agent for g_1 outcomes, but also for g_2 as well. These payments are, however, indirectly obtained from the "large" marginal payments made to the agent by group A. By scrutinizing expressions (44) and (44a), we see that these marginal payments to the agent by group A far outweigh the group's marginal valuations of the outcomes related to each of the agent's actions, or in short: $P_1^{A,POS} >> \Phi_1^A$ and $P_2^A >> 0$. Accordingly, in order for type A principals to achieve any benefit from organizing within the common agency framework, the group must either receive a fixed payment from the agent (a negative Q^A), or pay less in terms of their portion of the agent's fixed fee (a lower Q^A), as compared to either the unorganized sequential move case, or if the first-mover advantage is somehow an endogenous choice faced by type A principals, the partially organized simultaneous move case.[11] However these variable fees serve a dual purpose. Not only do they adjust the aggregate variable fees received by the agent so that the agent will implement the second best level of effort, but they also guarantee the participation of the agent by ensuring the participation of type B principals. If, in the sequential move game, partial cooperation is individually beneficial to type A principals, it is possible to implement the second-best outcomes without the aid of intervention by a central authority.

To check whether this is the case, we need to compare group A's welfare under the partially organized sequential move scenario with that obtained in the unorganized sequential move game. But before doing so, consider the fact that we can compare group A's welfare under the partially organized sequential move game with that which the group receives in the partially organized simultaneous move game. The argument can be made as follows. Given that type B principals respond in a Cournot-Nash fashion to group A's choice of fees to the agent, group A can achieve the same level of welfare by choosing to play the same strategies in the sequential move game as in the simultaneous move game. However, given that we observe group A choosing to implement different strategies in the leader-follower game, it must be the case that the group and therefore type A principals are, at least, individually no worse off then they would be in the simultaneous move game. Thus we can state the following result,

Result 1: *The sequential move game yields the same or higher level of utility to group A than what they would obtain in the simultaneous move game.*

We now turn to the case where both types of principals remain unorganized but type A principals retain their first-mover advantage vis-à-vis type B principals. After deriving results for this case, we can then determine whether type A principals are better off cooperating than not in this game of sequential moves. Once having done this, we attempt to extend our results to see if we can say anything about A principals' welfare if the very act of organizing confers the first-mover advantage.

2.3.1 Common Agency with Sequential Moves

The structure of this game is similar to that described as above. The only real difference is that each type A principal, though still retaining the ability to move first, no longer acts jointly with other type A principals. Instead each type A principal, in the first stage, takes the others' contributions as given while anticipating the aggregate responses of type B principals and the single agent. Thus in this noncooperative or unorganized sequential move game, Ai's problem is to choose Q^{Ai}, P_1^{Ai}, and P_2^{Ai} in order to

$$\text{Maximize} \quad \Phi_1^{Ai} a_1 + I^{Ai} - P_1^{Ai} a_1 - P_2^{Ai} a_2 - Q^{Ai}$$

$$\text{subject to} \quad \Phi_2^{B} a_2 + I^{B} - P_1^{B} a_1 - P_2^{B} a_2 - Q^{B} \geq 0$$

$$a_j = a_j(P_j) \text{ and } P_j^B = P_j^B(P_j^A) \; j = 1,2$$

and equation (41).

Combining equation (41) with B's participation constraint, solving for Q^{Ai}, and then plugging the result into Ai's objective function we can rewrite the maximization problem as

$$\text{Maximize} \quad \Phi_1^{Ai} a_1 + \Phi_2^{B} a_2 + I^{Ai} + I^{B} + Q^{A(-i)} + P_1^{A(-i)} a_1 + P_2^{A(-i)} a_2$$

$$- (1/2) r[\sigma_{11} P_1^2 + \sigma_{22} P_2^2] - C(a_1, a_2)$$

$$\text{subject to} \quad a_j = a_j(P_j) \text{ and } P_j^B = P_j^B(P_j^A) \; j = 1,2 .$$

Following similar procedures as before, we can obtain the resulting first-order conditions,

(48) $$P_1^{Ai}: \quad \Phi_1^{Ai} - P_1^{Ai} - r\sigma_{11} c_{11} P_1 = 0$$

(48a) $$P_2^{Ai}: \quad \Phi_2^{B} - P_2^{Ai} - r\sigma_{22} c_{22} P_2 = 0 .$$

For future reference, note that the presence of the term Φ_2^B in (48a) will ensure that the outcome of this game will differ from the simultaneous unorganized common agency game (see equation (12a)). Summing these conditions over all n^A results in

(49) $$\Phi_1^{A} - P_1^{A} - n^A r\sigma_{11} c_{11} P_1 = 0$$

(49a) $$n^A \Phi_2^{B} - P_2^{A} - n^A r\sigma_{22} c_{22} P_2 = 0 .$$

Breaking up the term P_1 into P_1^A and P_1^B in (49), and plugging the expressions for P_1^B, we use the resulting expression to solve for P_1^A. Following similar procedures in (49a) gives the expression for P_2^A. Using these results, we can then solve for P_1^B and P_2^B. Label these solutions with a superscript "US" to denote the unorganized common agency problem with sequential moves:

(50) $$P_1^{A,US} = \frac{\Phi_1^A(1+n^B r\sigma_{11}c_{11})}{1+nr\sigma_{11}c_{11}}$$

(50a) $$P_2^{A,US} = \frac{n^A\Phi_2^B[1+(n^B-1)r\sigma_{22}c_{22}]}{1+nr\sigma_{22}c_{22}}$$

(50b) $$P_1^{B,US} = -\frac{\Phi_1^A n^B r\sigma_{11}c_{11}}{1+nr\sigma_{11}c_{11}}$$

(50c) $$P_2^{B,US} = \frac{\Phi_2^B}{1+n^B r\sigma_{22}c_{22}}\left[\frac{1+nr\sigma_{22}c_{22}-n^A n^B r\sigma_{22}c_{22}-n^A n^B(n^B-1)(r\sigma_{22}c_{22})^2}{1+nr\sigma_{22}c_{22}}\right].$$

Also by adding equations (50) and (50b) and (50a) together with (50c) we obtain expressions for P_1^{US} and P_2^{US},

(51) $$P_1^{US} = \frac{\Phi_1^A}{1+nr\sigma_{11}c_{11}}$$

(51a) $$P_2^{US} = \frac{(n^A+1)\Phi_2^B}{1+nr\sigma_{22}c_{22}}.$$

And rather than showing solutions for a_1^{US} and a_2^{US}, it is sufficient to remember that these are but simple increasing monotonic transformations of P_1^{US} and P_2^{US} respectively. Using these results we can also derive

(52) $$Q^{A.US} = -\Phi_2^B \frac{1}{c_{11}} P_1^{US} - I^B - P_1^{A.US} \frac{1}{c_{11}} P_1^{US} - P_2^{A.US} \frac{1}{c_{22}} P_2^{US}$$

$$+ (1/2)(P_1^{US})^2[r\sigma_{11} + \frac{1}{c_{11}}] + (1/2)(P_2^{US})^2[r\sigma_{11} + \frac{1}{c_{22}}].$$

By observing the expressions in (50), (50b), and (51), we see that these solutions are no different from those obtained under the standard assumptions of common agency, where principals all act separately and move simultaneously. Thus at least in dimension of g_1, type A principals choose to implement their Cournot-Nash strategy as opposed to choosing any other strategy given that type B principals are Stackelberg followers. However, type A principals do not play a similar strategy with regard to the agent's choice of task two. We also see from (50b) and (50c) that as in the partially organized, sequential move game, type B principals receive payments from the agent for both g_1 and g_2 outcomes. Given that type A principals choose a different level of payment at least with respect to g_2 outcomes in the sequential move game than that selected in the simultaneous move game (see the game as depicted in Chapter 1), we can state our second result,

Result 2: *The noncooperative sequential move game yields the same or higher level of utility to unorganized type A principals than what they would receive in the noncooperative simultaneous move game.*

Given these results, we can now check to see if type A principals actually have the incentive to cooperate and organize in the sequential move, common agency game.

2.3.2 Comparison

We start out by comparing the solutions derived in Section 2.3.1 to the second-best outcomes derived under partial cooperation. First, it is not to hard to see that $P_1^{POS} > P_1^{US}$ and

$P_2^{POS} > P_2^{US}$, and therefore $a_1^{POS} > a_1^{US}$ and $a_2^{POS} > a_2^{US}$. That is, in the sequential move game, the aggregate marginal payments are stronger and therefore the agent's efforts, greater, when the lesser number of principals organize than when individual members of both types act unilaterally and separately. Analyzing the marginal payments by group gives the following:

$$P_1^{A.POS} > P_1^{A.US} \qquad P_2^{A.POS} > P_2^{A.US}$$

$$\left|P_1^{B.POS}\right| > \left|P_1^{B.US}\right| \qquad \left|P_2^{B.POS}\right| > \left|P_2^{B.US}\right|.$$

From these we see that type A principals pay more as a group than when they act separately. We also see that type B principals receive higher levels of payments from the agent when type A principals are organized than when they are not, even though in either case, type B principals receive their reservation level of utility.

Also with regard to group A's share of the agent's fixed fees, it can be shown that $Q^{A.POS} < Q^{A.US}$. This holds whether type A principals receive more in terms of payments from the agent or has to pay less in fixed fees to the agent in the partially organized case as compared to the case where they act independently. Finally using the expressions below,

$$U^{A.POS} = \Phi_1^A a_1^{POS} + I^A - P_1^{A.POS} a_1^{POS} - P_2^{A.POS} a_2^{POS} - Q^{A.POS}$$

$$U^{A.US} = \Phi_1^A a_1^{US} + I^A - P_1^{A.US} a_1^{US} - P_2^{A.US} a_2^{US} - Q^{A.US}$$

it is possible to show that type A principals are better off when organized, and hence in the sequential move game, not only are there incentives for type A principals to organize, but by doing so, the second-best outcome is implemented without the aid of a central authority.

Proposition 3: *Given a first mover advantage, type A principals are collectively and individually better off for having organized than not.*

Argument: In order to prove the claim, we need to compare group A's welfare in the sequential move game under standard common agency assumptions with that obtained under partial cooperation. We thus need to demonstrate that the expression

$$U^{A.POS} - U^{A.US} = \Phi_1^A a_1^{POS} + I^A - P_1^{A.POS} a_1^{POS} - P_2^{A.POS} a_2^{POS} - Q^{A.POS}$$

$$-[\Phi_1^A a_1^{US} + I^A - P_1^{A.US} a_1^{US} - P_2^{A.US} a_2^{US} - Q^{A.US}]$$

is positive. By making the appropriate substitutions and after some grouping of like-terms, it is possible to show that

$$\Gamma_0\left[\frac{(1+r\sigma_{11}c_{11})(1+nr\sigma_{11}c_{11})^2}{2}+\frac{(1+r\sigma_{11}c_{11})^3}{2}-\frac{2(1+r\sigma_{11}c_{11})^2(1+nr\sigma_{11}c_{11})}{2}\right]+$$

$$\Gamma_1\left[\frac{(1+r\sigma_{22}c_{22})(1+nr\sigma_{22}c_{22})^2}{2}+\frac{(n^A+1)^2(1+r\sigma_{22}c_{22})^3}{2}-\frac{2(n^A+1)(1+r\sigma_{22}c_{22})^2(1+nr\sigma_{22}c_{22})}{2}\right]$$

where

$$\Gamma_0 = \frac{(\Phi_1^A)^2}{c_{11}}\frac{1}{(1+r\sigma_{11}c_{11})^2(1+nr\sigma_{11}c_{11})^2}$$

$$\Gamma_1 = \frac{(\Phi_2^B)^2}{c_{22}}\frac{1}{(1+r\sigma_{22}c_{22})^2(1+nr\sigma_{22}c_{22})^2}$$

is positive. Since type A principals, in aggregate are better off when organized, individual type A principals can be all made potentially better off. Therefore type A principals have incentive to organize and implement the second-best solution given the strategic incentive in doing so. ■

These arguments and results seem to run counter to those found in Sandler (1992) and Varian (1994). It was shown there in a game of voluntary contributions to a public good with perfect information, that public provision is the same or lower when players move sequentially rather than simultaneously. Here however, the first-mover advantage encourages type A principals as a group to implement the second-best solution. In a model more closely related to the one explored here by Buchholz, Haslbeck, and Sandler (1998), a question arises whether the amount of the public good provided in a Stackelberg equilibrium is greater or smaller than the fully noncooperative solution. Their model is also a game of voluntary contribution to a public good, but with perfect information, and includes the possibility of partial cooperation. In the last section of their paper, members of the coalition also move first when determining their contributions, taking into account the reactions of the noncooperators. But as the authors mention, their results are inconclusive and as the authors hypothesize, within the context of their model, if cooperation is marginally profitable, one would expect to see a higher level of public provision. To the extent that we can compare these models, this claim seems to be reinforced by results obtained in this section.

Finally using Proposition 3 and Result 2, we can string these results together to form another result,

Result 3: *Given that* $U^{A,POS} > U^{A,US}$ *and* $U^{A,US} \geq U^{A}$, *then* $U^{A,POS} > U^{A}$.

Thus, if obtaining the first-mover advantage is dependent on the choice of organizing, i.e. somewhat endogenous to the problem of partial cooperation (as argued in endnote eleven), there will be sufficient motivation for group A principals to form a coalition and improve upon the standard common agency, simultaneous move game. In fact, if these assumptions hold, the preferred and second-best outcome will always be implemented since the most likely group to organize can achieve a higher level of utility then possible by remaining unorganized. This is a rather strong result considering the fact that this can be accomplished without the aid of central intervention. Although this result should be tempered by the fact that we did not explicitly model the choice of roles here, we have shown it is in the interest of one group to organize and that this leaves the remaining principals no worse off than before. The passive role played by the former principals, then, is at least consistent with the outcome of the game.

2.4 Conclusion

Although the model is quite specific, some of the results obtained in Sections 2.1 and 2.3 are interesting. Given what is already known about the common agency problem, that full cooperation improves upon the fully noncooperative case, one might expect that partial cooperation would at least have the potential of improving efficiency within the specified model. However as argued in the simultaneous move case, there appears to be no inducement for the group considered most likely to form, to act and organize unilaterally. Despite what would be an improvement in efficiency, most of the benefits quite possibly accrues to the unorganized principals and, most likely, in maintaining the agent's participation. Nonetheless in the sequential move case, if it can be assumed that organization bestows a first-mover advantage, partial cooperation can actually result in an outcome that is second best. It was also shown that at least in certain cases, the rather negative result in Section 2.1 can be overturned when certain payments made to the agent are restricted. That is, in the restricted and simultaneous move case, it is possible that partial cooperation can occur and improve efficiency – at least in the organizing sector.

One drawback of the analysis is not that one group was assumed to organize and the other not – for if all organized, we would be back to fully cooperative case – but rather that partial organization was potentially postulated to be predicated upon the group considered most likely to organize. Although results from Olson's *Logic of Collective Action* were used to argue the selection, the process seems somewhat arbitrary. However, given the structure of the model, it may be more sensible to address these issues within another context or with more amiable models. These topics are certainly important enough to warrant pursuit in the future.

Finally, it was argued earlier that in examining the case for central intervention, we were not just setting up a straw man. It should be also added that we were not just interested in showing that it could be done – restoring efficiency – for it may invariably be the case. But rather the interest in presenting such a scenario was to inquire whether this could be done in a somewhat decentralized fashion. There it was implied in a heuristic or slightly loose fashion that the requirements needed to achieve a second-best outcome might appear to be too high, that it might be best if the central authority were to limit itself to restricting certain payments to the agent while perhaps encouraging cooperation among the principals. On the other hand, there might be a weakness in this approach if the authority cannot adequately monitor these payments, since as was shown, partial cooperation can be self-defeating from the organizing group's point of view in the simultaneous move case. As for the sequential move game, it was demonstrated that there was no need for central intervention. In fact, if the central government placed restrictions on certain payments, this would actually inhibit type A principals from implementing the second-best outcome. Thus central intervention in this case would probably only make things worse. The foregoing analysis therefore may best be viewed as a cautionary tale for collective action and intervention within a common agency environment.

2.5 Notes

[1] A slightly similar argument is made by Becker (1983), where it is argued that smaller groups tend to be more politically successful than larger groups.

[2] When we talk of improved efficiency, we mean movements toward second-best outcomes in terms of the agent's marginal incentives and levels of effort.

[3] For example, expression (26) is derived by substituting in expressions for a_1, a_2, P_1^A, and P_2^A (or equations (15), (15a), (14), and (14a) respectively) into group A's aggregate utility function. Similar substitutions are made, for example, to find the expression for $U^{A,PO}$.

[4] The assumption of equal cost sharing is made in order to impose some structure on the problem so that the determination of the distribution of the agent's fixed fee can be made. One reason that the assumption is not imposed at the outset is that it slightly alters some of the results and makes others unattainable or noncomparable to results already existing in the literature.

[5] Instead of using the rule of equal cost sharing we could argue that group A's bargaining power actually weakens and B's strengthen relative to the standard unorganized common agency case using Olson's hypothesis of the exploitation of the large by the small (Olson, 1965). Despite the fact that each type of principal benefits separately in terms of the good or service they receive from the agent's actions, both types benefit from ensuring the agent's participation, and the aggregate fixed fee represents part of the costs of engaging the agent. And it is the splitting of this cost which the principals must bargain over or determine via some other method. In both cases, whether type A principals are unorganized or organized, they are always acting in relation to unorganized type B principals. In the former case when both types are unorganized, we must assume that sufficient agreement or structures (such as cultural norms) exist so as to independently determine the burden of this cost. In the latter case, type A principals act as one large principal in relation to many small type B principals. The effects of the organized group's actions now weighs greatly with respect to the outcome than (do) individual decisions by the remaining unorganized principals. In short, the organized group identifies more with the overall outcome and is more likely to ensure the engagement of the agent and getting her to provide them with the public good or service even though once engaged, the agent also provides similar services to others. Unorganized principals if they are aware of this, are then more likely to free ride then they would if type A principals were not organized. It is the increased likelihood of this threat and the awareness on the part of type B principals for using it that could cause a relative shift in bargaining power. We should emphasize and qualify the word "relative" since A may contribute more in both cases – the argument is that A pays more as a group when organized then when not. This argument would not change the results of Proposition 1 for type A principals but would for type B principals. If the argument holds, type B principals are unequivocally better off when A organizes than when they do not.

[6] Given the fact that the bracketed terms in the expression for group A's utility comparison are positive as long as $(n^B)^2 > n^A$, the above results hold even when the number of type A principals are greater than n^B. In the event that the bracketed terms are negative, type A principals may still be no better off organizing than not if the difference $(Q^A - Q^{A,PO})$ is sufficiently positive to outweigh the (now) two negative terms in the aforementioned expression.

[7] If as argued in endnote 5, group A's bargaining power weakens and Bs' strengthens, the result for type A principals remains ambiguous but type B principals are unequivocally better off whenever type A principals organize.

[8] Where, for example, the superscript A(-i) on the marginal payment for the first output represents the marginal payments paid by all A-type principals except for principal Ai.

[9] Note that type A and Type B aggregate resource constraints have already been substituted into their respective utility functions.

[10] Type B's participation constraint will be binding at the optimum.

[11] If the very act of organizing confers a first-mover advantage as a result of improved communication and coordination amongst the principals that not only allows the group to act decisively sooner but correctly anticipate the responses of those who remain unorganized, then in a sense, the choice of forming a coalition also simultaneously includes the choice of moving first.

CHAPTER 3: CLUBS AND COST SHARING

3.1 Introduction

Club theory has a relatively long history and has helped bridge the theoretical gap between private and pure public goods. It has therefore provided a useful framework for addressing a wide variety of issues (Glazer, Niskanen, and Scotchmer (1997)). This chapter follows in tradition, while focusing attention on the potential relation between membership size and the concept of cost sharing in clubs. Although the important relationship between membership and provision has been extensively studied and used, see Cornes and Sandler (1996) for an example of the former, and Sterbenz and Sandler (1992) for the latter, most studies have concentrated their attention on the effect of various alternative assumptions and scenarios on the level of club provision. Few studies have exclusively examined the impact of changes in a club's cost structure on membership. Oakland (1972) and Sandler and Tschirhart (1997) are, nonetheless, some exceptions.

The purpose of this chapter then is to readdress this issue. The argument for the possibility of a relationship between a club's costs and membership is somewhat intuitive since as a club's cost rises, then so do the potential benefits of apportioning these costs out among an increased membership. The reverse may hold true if a club's costs fall. Admittedly these statements need to be more precisely qualified, preferably in well-defined models, but the intuition remains. Invariably exceptions are possible and if not exceptions, then at least inconclusive results, but in the models to be presented in this chapter, this posited relationship will provide the common thread for the understanding of our results. In the next section, we investigate the impact of agency costs resulting from asymmetric information on the size of a given club's membership. In another setting, in Section 3.3, we look at the impact of an additional source of revenue that can reduce the club's need for cost sharing among a given number of club users. The final section concludes.

3.2 The Principal-Agent Problem: An Application to Clubs

It is well known that it is difficult to obtain but a few general results from even the simplest of hidden-action models (see for example, Grossman and Hart (1983), or for a less technical presentation and discussion, Salanie (1997)). However, with the appropriate assumptions, it can be shown in the case of the two-effort model, where each of the two levels of effort is associated with two possible outcomes, that it is relatively more costly to implement the higher level of effort when it is not observable, than when it is. The interpretation of this result can be explained as follows. Given the binding agent's participation constraint under both scenarios, the agent receives the same level of utility in either case. Yet because of the inability to directly contract on agent effort when it is not observable, it is essential, from an incentive point of view, to get the risk-averse agent to take on some risk. However, it then becomes necessary to compensate the agent with a higher level of expected payment than is needed to implement the same level of effort under conditions of perfect observability. In this latter case, the agent's payment can be fixed since it can be directly tied to the level of effort. Knowing the impact on the principal's costs to implement this (possibly) desired level of effort, we can then verify the prediction by Sandler and Tschirhart (1997) that the increased cost of agency can lead to larger club membership.

3.2.1 The Model

In order to demonstrate the above hypothesis we start by making the first simplifying assumption that potential club users, the principals in our model, are homogenous with respect to their tastes and incomes. It is also assumed that they agree to band together to design the agent's contract for implementing the desired level of effort and to jointly determine a common variable of interest (i.e. club membership). This is accomplished by agreeing to maximize the utility of a representative member subject to the relevant principal-agent constraints. One can think of this model as depicting the situation of a member-owned club with the agent as the manager of a club-like facility where the manager's actions, of say, the running of a complex of condominiums or safeguarding a gated community or a local area of business, are hidden.

As mentioned above, there are only two available levels of effort or actions from which the agent can choose. To streamline the presentation, we will only be concerned with the unified principals implementing what they consider to be the possible "best" action.[1] In addition, associated with each level of effort there are only two possible outcomes: a high (h) and a low (l) outcome. That is, in choosing one of the two levels of effort, a "best" level of effort and an "alternative" one, the agent influences the probability of each outcome. Let π_{ik} be the probability of outcome i, i = l, h given the k^{th} action, where k = a, b and $\pi_{lk} + \pi_{hk} = 1$. It is also assumed that action b stochastically dominates action a. This implies, here with strict inequality, that $\pi_{hb} > \pi_{ha}$ and $\pi_{lb} < \pi_{la}$. It should be noted that for more complicated models, additional assumptions such as the monotone-likelihood ratio property and the concavity of the distribution function condition, are sometimes made. But since there are only two outcomes, these assumptions here would be redundant (see Kreps (1990), p 600 for details). As will be seen, the reason for utilizing stochastic dominance is to ensure that the level of payment to the agent increases with the level of the outcome. Finally, assume that $C_b > C_a$, that the fixed cost of implementing action b is more costly to the agent, then implementing alternative action a.

The case to be analyzed is the standard one: of risk-neutral principals (with respect to the monetary payments to be made to the agent) and a risk-averse agent where the level of club provision, g_i, is random to both parties. For each possible outcome i, a payment t_i is specified by the principals to be paid to the agent. The representative club member's von Nuemann-Morgenstern (vNM) utility function has the following quasilinear form

(1) $$u = U(g_i, n) + W_i$$

where W_i represents the net wealth of the principal for outcome i. It is assumed that U(.) is twice differentiable and concave in both arguments, increasing with respect to g_i, and decreasing with respect to n. The expression for net-wealth in state i is defined as

(2) $$W_i = w - t_i/n$$

where w is the representative member's total wealth, independent of the possible states of the world, and the expression t_i/n, represents the cost sharing agreement among the principals, i.e., they share the expenses of the incentive scheme designed for the agent equally. Note that t_i is set before g_i is realized, so that the level of payment is made contingent on the possibility of a certain event occurring. It should also be mentioned that even though n represents the number of users of the club good, n is treated as a continuous variable as is standard procedure in the club literature. Lastly, the agent's vNM utility function is assumed to be additively separable and is given by

(3) $$V = v(t_i) - C_k$$

for i = l, h and k = a, b, where v(.) is strictly concave, twice continuously differentiable, and strictly increasing in t_i.

The representative principal's maximization problem then is to maximize expected utility given that the agent is to exert effort level b, subject to the agent's participation and incentive compatibility constraints. That is, the principals agree to choose t_h, t_l, and n, to

(4) $$\text{Maximize } \pi_{hb}[U(g_h, n) + w - \frac{t_h}{n}] + \pi_{lb}[U(g_l, n) + w - \frac{t_l}{n}]$$

$$\text{subject to } \quad \pi_{hb} v(t_h) + \pi_{lb} v(t_l) - C_b \geq 0$$

$$\pi_{hb} v(t_h) + \pi_{lb} v(t_l) - C_b \geq \pi_{ha} v(t_h) + \pi_{la} v(t_l) - C_a .$$

We first examine the case where there is no incentive problem, where effort is perfectly observable and then, the case where effort is not observable.

3.2.2 Observable Effort

When effort is observable, there is no need for the incentive compatibility constraint. The problem then can be seen as choosing t_h, t_l, and n to

(5) $$\text{Maximize } \pi_{hb}[U(g_h, n) + w - \frac{t_h}{n}] + \pi_{lb}[U(g_l, n) + w - \frac{t_l}{n}]$$

$$\text{subject to } \quad \pi_{hb} v(t_h) + \pi_{lb} v(t_l) - C_b \geq 0 .$$

Since it is possible to lower payments to the agent and still obtain the agent's participation as long as the constraint is satisfied, the participation constraint will be binding at the optimum. Using λ as the Lagrange multiplier, the first-order conditions are

(6) $$t_i : \quad \frac{1}{nv'(t_i)} = \lambda \quad i = h, l$$

(6a) $$n: \quad \pi_{hb}[U_n(g_h, n) + \frac{t_h}{n^2}] + \pi_{lb}[U_n(g_l, n) + \frac{t_l}{n^2}] = 0$$

(6b) $$\lambda: \quad \pi_{hb} v(t_h) + \pi_{lb} v(t_l) - C_b = 0 .$$

Using these conditions, we can easily obtain the results shown below.

Result 1: Observable effort

(1.i) $t_h = t_l = t$

(1.ii) $v(t) = C_b$

(1.iii) $-[\pi_{hb} nU_n(g_h, n) + \pi_{lb} nU_n(g_l, n)] = \frac{t}{n}$.

Together, the two conditions from (6) imply (1.i), that the agent receives a fixed payment in return for implementing effort level b. This result can then be applied to the remaining first-order conditions to obtain the remainder of the results. Result (1.ii) states that the agent's expected surplus is set equal to zero, equal to the agent's normalized level of reservation utility. Result (1.iii) is the membership condition where the membership fee, t/n, is equated to the expected cost of congestion that the additional principal or member imposes on others. These are standard agency and club results.

3.2.3 Effort is Not Observable

Now suppose that there is an incentive problem, then the incentive compatibility constraint must be included into the principal's maximization problem. The problem then is as exactly stated earlier, as in (4). The first-order conditions are

(7) t_h : $\frac{1}{nv'(t_h)} = \lambda + \mu[1 - \frac{\pi_{ha}}{\pi_{hb}}]$

(7a) t_l : $\frac{1}{nv'(t_l)} = \lambda + \mu[1 - \frac{\pi_{la}}{\pi_{lb}}]$

(7b) n: $-[\pi_{hb} nU_n(g_h, n) + \pi_{lb} nU_n(g_l, n)] = \pi_{hb}\frac{t_h}{n} + \pi_{lb}\frac{t_l}{n}$

(7c) λ: $\pi_{hb} v(t_h) + \pi_{lb} v(t_l) - C_b = 0$

(7d) μ: $\pi_{hb}v(t_h)+\pi_{lb}v(t_l)-C_b = \pi_{ha}v(t_h)+\pi_{la}v(t_l)-C_a$

where λ and μ are the Lagrange multipliers associated with the participation and incentive compatibility constraints, respectively. From these first-order conditions, the following results are derived. The proof of result (2.i) is given in the appendix. The other two are restatements of the first-order conditions (7c) and (7b), shown here, for convenience.

Result 2: Effort is Not Observable

(2.i) $t_h > t_l$

(2.ii) $\pi_{hb}v(t_h)+\pi_{lb}v(t_l) = C_b$

(2.iii) $-[\pi_{hb}nU_n(g_h,n)+\pi_{lb}nU_n(g_l,n)] = \pi_{hb}\frac{t_h}{n}+\pi_{lb}\frac{t_l}{n}$.

Given the earlier assumptions, payment to the agent is monotone with respect to the outcome, and thus under imperfect observability with a risk-averse agent, there is risk sharing between the principals and the agent. This is the standard agency result. Result (2.ii) states that the agent's expected surplus is again set equal to zero, the normalized level of reservation utility, and (2.iii) is the membership condition where the right-hand side represents the expected membership fee.

3.2.4 Comparison

It is also of interest to compare the variables of central concern under the two regimes. To facilitate the discussion, let any variable with a single "*" denote a solution to the principal's problem when actions are observable, and let a double asterisk, "**", denote a solution under the hypothesis that actions are not observable. To start, note that when there is an incentive problem, because of the additional constraint required, a principal's expected utility when implementing the best effort can be no higher than his expected utility when implementing the same level of effort when there is no incentive problem. The remaining results of the comparison are summarized below.

Result 3: Comparison of the Two Cases

(3.i) $\tau^{**} > \tau^{*}$ *where* $\tau = \pi_{hb} t_h + \pi_{lb} t_l$ *for given levels of* t_h *and* t_l

(3.ii) $n^{**} > n^{*}$.

The proofs for these results are in the appendix to this chapter. Result (3.i) states that the expected payment to the agent is higher when the agent's efforts are not observable than when the agent's efforts are – where the expected payment, τ^{*}, is the fixed payment to the agent, t^{*}. This represents the added cost to agency when an agent's actions are hidden. Finally, result (3.ii) is a result specific to clubs and to the club problem as laid out in this section. It states that when there is an incentive problem, membership will be higher than it would be otherwise. This result shows that the claim by Sandler and Tschirhart (1997), that agency costs raise the benefit of cost sharing and therefore leads to an expansionary bias in club membership, holds, at least for the present model. Given that higher membership leads to increased congestion, a higher level of n can be thought of as adding to the costs of agency, though this does not imply greater inefficiency or suboptimality since principals tradeoff the benefits to cost sharing with crowding costs. We now turn to a quite different model and explore how club membership may respond when the benefit to cost sharing among users is reduced.

3.3 The Potential Effect of Advertising on Membership Size

This section looks into the possible effect of an additional source of revenue, other than the charging of club fees, on membership size. This additional source of funds can potentially be acquired from many different areas, but it is assumed here that the source of these funds is advertising revenue that the club receives in return for allowing firms to advertise to its members. Advertising, for example may play a significant role in clubs that seek sponsorship for its activities or in congestible networks, such as the Internet, where service providers seek additional means to cover network costs.[2] As mentioned in the introduction, the addition of such revenue may actually serve to reduce the benefits of cost sharing, which therefore can lead to a reduction in club membership. Although such a hypothesis and result makes sense in a club framework, it seems somewhat paradoxical within a standard advertising context. However, this does not always have to be the case. Without crowding, indeed if a club has a more pure public good aspect to it, for example like television, this result may no longer hold since under such conditions, the network might desire as large an audience as possible. That is, as long as the costs of reaching them do not outstrip the benefits. Whether the following results still remain somewhat counter-intuitive or not, it is hoped that they at least point to the potential value of portraying clubs in a partial equilibrium setting and investigating the potential effects that may result from the club's strategic interaction with other players.

This section looks at two models that are almost at the polar ends of the spectrum in regard to the types of firms manufacturing the private good and providing advertising revenue to a club. In the first case, the private good, x, is produced by a monopolistically competitive industry while in the second scenario, it is manufactured by a monopoly. In both instances, it is assumed that both types of firms believe it to be in their interest to advertise since advertising serves to increase the demand for their product. The club for its part, offers an excludable public good and a means by which advertisers can simultaneously reach the buyers of its product. In either case, the club provider should be thought of as a monopoly provider of not only of the club good or service, but also the medium by which the advertising can be conveyed to its users and purchasers of the private good.

3.3.1 When x is Produced by a Monopolistically Competitive Industry

In this first case, the club provider's decisions are explicitly modeled while the monopolistically competitive industry's decisions are not.[3] Given the apparent discrepancy of power between these two industries, it is assumed that the club is acutely aware of its monopoly control of the advertising medium so that it is able to set the schedule of advertising rates it can charge. It is also assumed that the private good industry will respond to the schedule inelastically, in aggregate. Nonetheless, the club must be aware of the fact that it cannot simply and arbitrarily set its rates. That is, it will need to consider the effect of how the rates must not only reflect the benefits received by the users and members of the club, but also that they should be fairly based on the size of the receiving audience. Thus, the provider will, when choosing the amount of members to be allowed into the club, take into account how this decision impacts advertising revenue, network costs, and user well-being.

The chief aim of these assumptions is to try to simulate as closely as possible, the process by which some media firms appear to undergo when attempting to attract advertising dollars. The amount they set for advertising spots must, in some way, be realistically tied to the medium's potential audience or membership, as well as to their tastes and other demographics. However, the analogy is not completely perfect, since if we took the example of a congestible network here, there can be a downside for the network (and for users) in attracting too many users of the resource in question. This may not always be the case with other media.

The structure of the game is as follows. There are three sets of players of interest: homogenous user/consumers, advertising private industrial firms, and the club provider. The provider, however, is the only player who explicitly behaves in a strategic manner. Within the context of our game, this means that the club provider explicitly takes into account the effect of its choices on the actions of the other players. The game is broken down into three stages. In order to solve the game, we utilize the standard concept of subgame perfection. The three stages are:

Stage 1: The club provider simultaneously chooses the advertising schedule (or function), A(n), and the number of users, n, allowed access to the club;

Stage 2: The private industry responds by supplying the dollar amount of advertising revenue to the club for n users; and

Stage 3: A representative club member chooses his or her consumption bundle of private goods, x and y.

Note that due to our previous assumptions, the structure of the game can easily be represented as a two-stage game – it is only for the clarity of presentation that the game is broken down into three stages.

We thus start at the last stage of the game and analyze the representative user's utility maximization problem. Assume first that the user's utility is quasilinear in the competitively provided numeraire good, y, and separable with regards to the other private good, x, and the level of provision of the club good, g. Taking as given at this stage, the level of advertising, A, measured in dollars, and the access fee, f, along with the price of x, p, and the user's income I, the representative user's utility maximization problem is specified as

(8) $$\text{Maximize } U = y + h(x, A) + u(g)$$

$$\text{subject to} \quad y + px + f = I$$

where $h_x > 0$, h_{xx}, $h_{AA} < 0$, and $h_{xA} > 0$. For simplicity, it is assumed that the level of provision of club good, g, is fixed. This assumption can be seen as being consistent with conducting a short-run analysis of our partial equilibrium model. Also note that the derivative h_A is left unsigned. The reason for this will become apparent later on. After solving the budget constraint for y and substituting the resulting expression into the objective function, we can then derive the user's demand for the private good by maximizing the user's utility with respect to x. Since the utility function is linear in y and additively separable, the user's demand for x is a function of its own price and the total level of aggregate advertising, $x = x(p, A)$.

In stage two, it is assumed that the monopolistically competitive industry responds to the club's prior choices of A and n by supplying exactly the amount demanded by the club. Although not directly modeled, it is assumed that there are sufficient forces to put an upper limit on A so as to prevent exit from the x industry.

At the initial stage the club provider simultaneously chooses A(n) and n.[4] The club's objective is to maximize the utility of a representative user while covering all costs and taking into account the effect of its choices on both the private industry and the representative user's actions. One rationale for satisfying this objective is to ensure that all potential users have, at least, the incentive to join. That being the case, a user should at least receive a utility level no less than his or hers reservation level. Although a user's reservation utility, $\overline{U}$, is not specifically incorporated into the model in the form of a constraint, the results of the upcoming maximization problem must eventually be checked against it, to see if this is indeed, the case.

The club provider faces two types of costs, C(g, n) and Ψ(A). C(g, n) represents the typical club cost function where C(.) is increasing and convex in n. The cost function, Ψ(A), represents the costs incurred by the club, say, for designing or formatting, putting up, and/or storing the private industry's advertising messages and is assumed to be increasing in A. As in most of the club literature, exclusion is assumed to be costless. Also since the provider must cover its costs, it faces the budget constraint,

(9) $$A + nf = C(g, n) + \Psi(A) .$$

Solving for f and substituting the result into the representative user's indirect utility function resulting from the solution of the user's maximization problem from the third stage, we can now specify the club provider's optimization problem as choosing A(n) and n to

Maximize

$$I - px(p, A(n)) - \frac{[C(g, n) + \Psi(A(n)) + A(n)]}{n} + h(x(p, A(n)), A(n)) + u(g) .$$

The first-order conditions, before application of the envelope theorem, are then

(10) A(.): $$(h_x - p)\frac{\partial x}{\partial A} - \frac{(\Psi_A - 1)}{n} + h_A = 0$$

(10a) n: $$(h_x - p)\frac{\partial x}{\partial A}\frac{\partial A}{\partial n} + \left[-\frac{(\Psi_A - 1)}{n} + h_A\right]\frac{\partial A}{\partial n} - \frac{C_n}{n} + \frac{C + \Psi - A}{n^2} = 0 .$$

Using the representative user's first-order condition from stage three, we can rewrite (10) as

(11) $$nh_A + 1 = \Psi_A .$$

The left-hand side of (11) represents the total marginal benefits to the club (from the point of view of all users), from advertising – the first term being the sum of the marginal benefits to club users and the second, the dollar in cost savings enjoyed by all members. The right-hand side is just the marginal cost of incorporating advertising into the club structure.[5]

Through successive applications of the envelope theorem, equation (11) reduces down to

$$-\frac{C_n}{n}+\frac{C+\Psi-A}{n^2}=0.$$

Rewriting this expression as

$$\frac{C(g,n)}{n}-C_n(g,n)=\frac{A(n)-\Psi(A(n))}{n} \tag{12}$$

we can see, ceteris paribus, that the standard average cost term for a club, $C(g,n)/n$, will exceed the marginal cost, C_n, of adding another user, so long as $A(n) - \Psi(A(n)) > 0$ for all n, that is, as long as the network's advertising revenue always exceeds the costs of implementing the x-industry's advertising programs. In a similar club model without advertising, these two costs – the two terms on the left-hand side of (12) – would normally be equalized at the optimum and hence, the conclusion that membership will be lower with advertising than without.

This conclusion, again, may seem paradoxical in that advertising payments made by the private industry may result in a decrease in club membership (though this is not quite the same thing as saying it results in a reduced audience). This is partly a result of the institutional and market structure (respectively) of the club and the private industry. Although this may not always be the case, but it is true here, payments made to an institution that engages in cost sharing, decreases that institution's need to attract more members. As regards to market structure, it was taken for granted that the x industry automatically responds with a flow of money equal to the club's demands for it, without taking into account the effect that the flow of payments has on membership. With this in mind, we alter the institutional and market structure and turn to the case where x is produced by a monopolist who moves first and who has the power to determine the level of advertising expenditure. These assumptions enable monopolist to incorporate the club's responses in membership to changes in its choices of advertising.

3.3.2 When x is Produced by a Monopolist

Using the concept of subgame perfection, we start with the club provider's decision problem after it has already allowed for the representative user's optimum choice of x at the last or third stage of the game. Again the level of club provision, g, is assumed to be held constant, and in order to simplify matters a bit, we also assume that the club incurs no costs in implementing the monopolist's desired level of advertising. At the second stage then, the club's problem is to choose n in order to maximize the utility of a representative user, or

$$(13) \qquad \text{Maximize} \qquad I - px(p, A) - \frac{[C(g, n) - A]}{n} + h(x(p, A), A) + u(g).$$

This leads to the following expression,

$$(14) \qquad nC_n(g, n) - C(g, n) + A = 0$$

which is similar to the result obtained above, $C/n - C_n = A/n$. Solving for n, we obtain n = n(g, A), the club's optimal response to the monopolist's choice in the level of advertising. Using the implicit function theorem, we then can derive the following comparative static from (14): $\partial n/\partial A = -1/(nC_{nn}) < 0$, which represents the effect of the monopolist's choice of A on club membership. From this comparative static, it is immediate that membership is inversely related to changes in the club's advertising revenues.

At the first stage then, we assume that x can be produced at constant cost, that there are no fixed costs, and that the monopolist incurs a cost of α, per unit of the advertising dollar. The monopolist's problem is to maximize profits while choosing p and A while anticipating the responses of the club and users who consume x, or

(15) Maximize $\pi = (p - c)n(g, A)x(p, A) - \alpha A$.

From which the following first-order conditions are derived,

(16) p: $nx + (p - c)n\left(\frac{\partial x}{\partial p}\right) = 0$

(16a) A: $(p - c)\left(\frac{\partial n}{\partial A}\right)x + (p - c)n\left(\frac{\partial x}{\partial A}\right) - \alpha = 0$.

We analyze each condition separately. Multiplying (16) by p/x and rearranging slightly gives

(17) $$\frac{p - c}{p} = \frac{1}{-\eta_{xp}}$$

where $\eta_{xp} = (\partial x/\partial p)(p/x)$. This condition, that the price-cost margin is set equal to the reciprocal of the price elasticity of demand, is the familiar Lerner index of monopoly power. Turning to condition (16a), now written as expression (18) below, it is informative to interpret the condition before rewriting it in terms of elasticities:

(18) $$(p - c)n\left(\frac{\partial x}{\partial A}\right) = \alpha - (p - c)\left(\frac{\partial n}{\partial A}\right)x .$$

The left-hand side of (18) represents the increase in the monopolist's profits due to an increase in advertising and is a result of a rightward shift in the aggregate demand curve of x.[6] This however, must not only be set against the per unit cost of advertising, but also by the membership response of the club to the increase in its advertising revenue. From the previous comparative static, $\partial n/\partial A$, as advertising increases, there is reduced incentive to retain membership at its current levels. This effect, reduces the advertiser's targeted audience. Finally, rewriting (18) in elasticity form, we obtain

(19) $$(p-c)[\eta_{xA}+\eta_{nA}]=\frac{\alpha A}{nx}$$

where $\eta_{xA} = (\partial x/\partial A)(A/x)$ and $\eta_{nA} = (\partial n/\partial A)(A/n)$. Using (17) we can rewrite the above condition as

(20) $$\frac{\eta_{xA}+\eta_{nA}}{-\eta_{xp}}=\frac{\alpha A}{pnx}.$$

Equation (20) states that the profit maximizing monopolist equates the ratio of its advertising costs to its sales revenue from club users to $(\eta_{xA}+\eta_{nA})/(-\eta_{xp})$, the ratio of the sum of the responses of users of x and club membership to a change in advertising expenditure by the monopolist, to the response of x to changes in its own price. Equation (20) is roughly comparable to Dorfman-Steiner result which states that a monopolist sets the ratio of advertising expenditure to sales revenue equal to the ratio of advertising elasticity of demand to the price elasticity of demand (Schmalensee, 1972). The difference in (20) then from the Dorfman-Steiner result, is the need for monopolist to also consider the impact of its advertising revenue on club membership.

3.4 Conclusion

We have looked at two distinct circumstances in which a club's cost structure underwent certain changes and have analyzed these effects on membership. Under the first scenario, agency costs resulting from the principal-agent problem lead to increased membership size due to the increased benefits of engaging in further cost sharing among a larger membership base. In the second case, we examined the impact of an additional source of club revenue used for covering club costs, and have shown that this can reduce the need for cost sharing among a pre-existing membership. Although each case is special, both demonstrate the usefulness of utilizing the concept of cost sharing when analyzing certain changes in the structure of the club itself or in its environment.

3.5 Appendix

Proof of Result (2.i):

When actions are not observable, we first need to show that both constraints are binding at the optimum, i.e., that $\lambda, \mu > 0$. We start by first assuming that $\lambda = 0$. Given $\pi_{lb} < \pi_{la}$, $\mu \geq 0$, and that we are assuming an interior solution, then in order for (7a) to hold, we need $v'(t_l) \leq 0$, which is contrary to our assumption of positive marginal utility. Now suppose that $\mu = 0$. Then from (7) and (7a) we have: $t_h = t_l = t$. But this violates the agent's incentive compatibility constraint since substituting t into

$$\pi_{hb} v(t_h) + \pi_{lb} v(t_l) - C_b \geq \pi_{ha} v(t_h) + \pi_{la} v(t_l) - C_a$$

results in $C_b \leq C_a$. A contradiction, so $\mu > 0$.

Finally we can now proceed and obtain our result. Subtracting (7a) from (7) gives

$$\frac{1}{nv'(t_h)} - \frac{1}{nv'(t_l)} = \mu\left[\frac{\pi_{la}}{\pi_{lb}} - \frac{\pi_{ha}}{\pi_{hb}}\right].$$

Using $\pi_{hb} > \pi_{ha}$ and $\pi_{lb} < \pi_{la}$, we obtain

$$\frac{1}{nv'(t_h)} > \frac{1}{nv'(t_l)}$$

or $v'(t_h) < v'(t_l)$, or $t_h > t_l$.

Proof of Result (3.i):

Because of binding participation constraints under both scenarios, we can show that expected payment for the agent's best effort is greater when effort is not observable than when it is. From these two constraints

$$v(t^{*}) = C_b$$

$$\pi_{hb} v(t_h^{**}) + \pi_{lb} v(t_l^{**}) = C_b ,$$

we obtain

$$v(t^{*}) = \pi_{hb} v(t_h^{**}) + \pi_{lb} v(t_l^{**}) .$$

Using Jensen's inequality: $Ev(t) < v(Et)$ if $v'' < 0$, we can then write

$$\pi_{hb} v(t_h) + \pi_{lb} v(t_l) < v(\tau)$$

where $\tau = Et = \pi_{hb} t_h + \pi_{lb} t_l$. More specifically,

$$v(t^{*}) = \pi_{hb} v(t_h^{**}) + \pi_{lb} v(t_l^{**}) < v(\tau^{**})$$

or $t^{*} < \tau^{**}$.

Proof of Result (3.ii):

In order to show that club membership can be higher when agent effort is not observable, we first draw some implications from concavity of $U(g_i, n)$. Let

$$EU(n) = \pi_{hb} U(g_h, n) + \pi_{lb} U(g_l, n)$$

$$EU_n(n) = \pi_{hb} U_n(g_h, n) + \pi_{lb} U_n(g_l, n) .$$

So by definition of concavity,

(A1) $$EU(n^{**}) \leq EU(n^{*}) + EU_n(n^{*})(n^{**} - n^{*})$$

(A2) $$EU(n^{*}) \leq EU(n^{**}) + EU_n(n^{**})(n^{*} - n^{**}) .$$

Adding and rearranging (A1) and (A2),

$$0 \le \Delta n[EU_n(n^*) - EU_n(n^{**})]$$

where $\Delta n = n^{**} - n^*$. Then if

(A3) $$\Delta n \le 0, \qquad EU_n(n^*) \le EU_n(n^{**})$$

$$\Delta n > 0, \qquad EU_n(n^*) > EU_n(n^{**}).$$

Now turning to the membership conditions under the two scenarios,

$$-[\pi_{hb} n^* U_n(g_h, n^*) + \pi_{lb} n^* U_n(g_l, n^*)] = \frac{t^*}{n^*}$$

$$-[\pi_{hb} n^{**} U_n(g_h, n^{**}) + \pi_{lb} n^{**} U_n(g_l, n^{**})] = \pi_{hb} \frac{t_h^{**}}{n^{**}} + \pi_{lb} \frac{t_l^{**}}{n^{**}},$$

we can rewrite these as

(A4) $$-n^{*2} EU_n(n^*) = t^*$$

(A5) $$-n^{**2} EU_n(n^{**}) = \tau^{**}.$$

From (A4) and (A5), and using the result $\tau^{**} > t^*$, we can obtain the following inequality,

(A6) $$\frac{n^{**2}}{n^{*2}} > \frac{EU_n(n^*)}{EU_n(n^{**})}.$$

Now suppose that $n^{*2} \ge n^{**2}$, i.e. $n^* \ge n^{**}$ or $\Delta n \le 0$. Then from (A6),

$$1 > \frac{EU_n(n^*)}{EU_n(n^{**})} > 0$$

or $EU_n(n^{**}) < EU_n(n^{**})$, a contradiction of one of the results, (A3), derived under the assumption of concavity. Thus $n^{**} > n^*$.

3.6 Notes

[1] We ignore the implementation of the other "alternative" action. To fully complete the analysis, one would have to compare the indirect utility of the representative principal under both effort levels and the number of club members associated with each. However, for a given membership size, the level of effort that gives the representative principal the highest level of utility would be the most desirable level of effort that the principals would like to implement.

[2] For a rationale of treating network resources as congestible goods or resources, see MacKie-Mason and Varian (1994).

[3] With regard to not explicitly modeling the x industry, this may seem a bit unsatisfactory to some, yet this is done to simplify the exposition of our main concern: an investigation into the impact of changes in a club's cost structure on membership. However, we should mention that what we have in mind is a private good industry with many firms selling differentiated products. Sufficiently so as to warrant advertising, but only slightly enough so, as to define industry output and advertising as the sum of individual outputs and money spent by individual firms. Impacts of changes in individual firm behavior are also assumed to be significantly small enough such that each firm ignores the effect of its actions on others.

[4] In a more formal presentation, one may want to limit the set of feasible advertising functions from which the club provider can or would choose from. For example, if A is an element of a set of functions S, then $A(n) \geq 0$ for all n in a population of N candidates for membership. That is, the club requires that it receive non-negative advertising revenue. We may also want to define a permissible range of the functions, say that $0 \leq A(n) \leq A^{\circ}$ for all $n \in N$.

[5] Other things being equal, the term h_A will be positive here only if, at the margin, $\Psi_A > 1$. That is, the marginal cost of implementing the desired level of advertising is greater than the marginal cost savings of receiving the advertising revenue. If on the other hand, there are minimal advertising implementation costs, say when we have $nh_A + 1 = 0$, then $h_A < 0$. Members would then be on the negative portion of their aggregate marginal benefit curve so as to fully capture the cost savings enjoyed by the additional outside source of revenue.

[6] Using the first-order condition of the representative user's utility maximization problem and the implicit function theorem, $p - h_x(x(p,A), A) \equiv 0$, we obtain $\partial x/\partial A = -(h_{xA}/h_{xx}) > 0$.

CONCLUSION

In models of voluntary provision of public goods, it is a well-established fact that unilateral action to contribute to public goods by an individual while facing all the costs, is unlikely. In Mancur Olson's view, there is no getting around this result. Underprovision of public good can only be overcome through the use of selective incentives, coercion, or institutional arrangement. Since Olson's contributions, it has been fashionable to find ways that overturn this result, and under the right assumptions, this, no doubt can be accomplished.

The first two chapters concerning common agency and partial cooperation, though utilizing a slightly different perspective and approach, followed a similar story line. A modified common agency model was used to investigate the impact of partial cooperation on the construction of agent incentives and effort in providing two collective goods. These goods had the special characteristics that each was independently related to the agent's actions but were public to one type of principal and private to the other.

In addition, partial cooperation was introduced into the model through the assumption that only one type of principal (the smallest homogenous group of principals) was likely to form a coalition. Introducing cooperation in this manner allowed us to assume that some of the incentives for free riding had been overcome. Yet, in the simultaneous case, we saw from the organizing principals' perspective, that partial cooperation was self-defeating despite a strengthening of agent incentives and effort. In the sequential move game, it was shown that having a first-mover advantage over unorganized principals was not only individually beneficial to the cooperating principals, the outcome in terms of agent incentives and effort was constrained Pareto efficient, better than even the third-best, noncooperative outcome.

Thus, despite some moderately strong assumptions made along the way, these results demonstrate that partial cooperation can, but need not always be the panacea for improving efficiency. Results often depend on institutional detail and individual incentives.

In addition, given the rather pessimistic result concerning partial cooperation in the simultaneous move game, we turned to investigating various remedies for improving efficiency. It was established by restricting principal punishments, that under certain circumstances, partial cooperation was capable of improving efficiency by delivering the second-best, but restricted outcome, at least for the organizing principals. We also looked at two other methods of achieving the second-best outcome. The first policy involved subsidizing the principals' marginal payments to the agent. But as shown, this method had the potential of being relatively wasteful in the use of the economy's resources. The second policy utilized a more direct approach, subsidizing the agent directly. The drawback in using this policy however, was that it required a bit more structure in terms of central planning and principal behavior.

Finally, in a separate essay, we studied the relation of cost sharing and membership size within the framework of club goods. The intuition for this relationship is fairly straightforward and rested on the idea that with a given membership base and everything else held constant, a rise in costs increases the benefits of cost sharing while a fall, reduces them. The third chapter shows that at least in certain cases, this intuition holds, and as a result, depending on whether the benefits rise or fell sufficiently, one would expect to see a resulting rise or fall in membership.

BIBLIOGRAPHY

Becker, Gary S. "A Theory of Competition Among Pressure Groups for Political Influence." *Quarterly Journal of Economics* 98 (August, 1983): 371-400.

Bernheim, B. Douglas, and Whinston, Michael. "Common Marketing Agency as a Device for Facilitating Collusion." *Rand Journal of Economics* 16 (Summer 1985): 269-281.

Bernheim, B. Douglas, and Whinston, Michael. "Menu Auctions, Resource Allocation, and Economic Influence." *Quarterly Journal of Economics* 101 (February 1986): 1-31.

Bernheim, B. Douglas, and Whinston, Michael. "Common Agency." *Econometrica* 54 (July 1986): 923-942.

Bond, Eric, and Gresik, Thomas. "Regulation of Multinational Firms with Two Active Governments: A Common Agency Approach." *Journal of Public Economics* 59 (January 1996): 33-53.

Buchholz, Wolfgang, Christian Haslbeck, and Todd Sandler. "When Does Partial Cooperation Pay?" *Finanzarchiv* 55 (1998), Forthcoming.

Cornes, Richard, and Sandler, Todd. *The Theory of Externalities, Public Goods, and Club Goods*, 2nd Ed. New York: Cambridge University Press, 1996.

Dixit, Avinash. *The Making of Economic Policy*. Cambridge, MA: MIT Press, 1996.

Dixit, Avinash. "Power of Incentives in Private versus Public Organizations." *American Economic Review (Papers and Proceedings)* 87, no. 2 (May 1997): 378-382.

Dixit, Avinash, Gene M. Grossman, and Elhanan Helpman. "Common Agency and Coordination: General Theory and Application to Government Policy Making." *Journal of Political Economy* 105 (August 1997): 752-769.

Dixit, Avinash, and Norman, Victor. "Advertising and Welfare." *Bell Journal of Economics* 9 (Spring, 1978): 1-17.

Dixit, Avinash, and Norman, Victor. "Advertising and Welfare: Reply" *Bell Journal of Economics* 10 (Autumn, 1979): 728-729.

Dixit, Avinash, and Norman, Victor. "Advertising and Welfare: Another Reply." *Bell Journal of Economics* 11 (Autumn, 1980): 753-754.

Fisher, Franklin M., and McGowan, John J. "Advertising and Welfare: Comment." *Bell Journal of Economics* 10 (Autumn, 1979): 726-727.

Glazer, Amihai, Esko Niskanan, and Suzanne Scotchmer. "On the Uses of Club Theory: Preface to the Club Theory Symposium." *Journal of Public Economics* Special Issue, 65 (July, 1997): 3-7.

Grossman, Gene M., and Helpman, Elhanan. "Protection for Sale." *American Economic Review* 84 (September 1994): 833-850.

Grossman, Sanford J. and Hart, Oliver D. "An Analysis of the Principal-Agent Problem." *Econometrica* 51 (January, 1983): 7-45.

Guttman, Joel. "Voluntary Collective Action." In *Markets and Politicians*, edited by Arye L. Hillman, 27-41. Boston: Kluwer Academic Publishers, 1991.

Hodge, B.J., William P. Anthony, and Lawrence M. Gales. *Organization Theory: A Strategic Approach*, 5th Ed. Upper Saddle River, NJ: Prentice Hall, 1996.

Hochman, Oded, and Luski, Israel. "Advertising and Economic Welfare: Comment." *American Economic Review* 78 (March, 1988): 290-296.

Holmstrom, Bengt, and Milgrom, Paul. "Aggregation and Linearity in the Provision of Intertemporal Incentives." *Econometrica* 55 (March 1987): 303-328.

Holmstrom, Bengt, and Milgrom, Paul. "Regulating Trade Among Agents." *Journal of Institutional and Theoretical Economics* 146 (1990): 303-328.

Holmstrom, Bengt, and Milgrom, Paul. "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design." *Journal of Law, Economics, and Organization* 7 (1991): 24-51.

Laffont, Jean-Jacques. *The Economics of Uncertainty and Information*. Cambridge, MA: MIT Press, 1989.

Mackie-Mason, Jeffrey K., and Varian, Hal R. "Pricing Congestible Network Resources." Mimeo, 1994.

Martimort, David. "Multiple Principals and Asymmetric Information." Ph.D. diss., Université de Toulouse, 1990.

Martin, Stephan. *Advanced Industrial Economics*. Cambridge, MA: Blackwell Publishers, 1993.

Mas-Colell, Andrew, Michael D. Whinston, and Jerry R. Green. *Microeconomic Theory*. New York: Oxford University Press, 1992.

Milgrom, Paul, and Roberts, John. *Economics, Organizations and Management*. Englewood Cliffs, NJ: Prentice Hall, 1992.

Nichols, Len M. "Advertising and Economic Welfare." *American Economic Review* 75 (March, 1985): 213-218.

Oakland, William H. "Congestion, Public Goods, and Welfare." *Journal of Public Economics* 1 (November, 1972): 339-357.

Olsen, Trond, and Torsvik, Gaute. "The Ratchet Effect in Common Agency: Implications for Regulation and Privation." *Journal of Law, Economics, and Organization* 9 (1993): 133-158.

Olsen, Trond, and Torsvik, Gaute. "Intertemporal Common Agency and Organizational Design: How Much Decentralization?" *European Economic Review* 39 (August 1995): 1405-1428.

Olson, Mancur. *The Logic of Collective Action*. Cambridge, MA: Harvard University Press, 1965.

Salanié, Bernard. *The Economics of Contracts: A Primer*. Cambridge, MA: MIT Press, 1997.

Sandler, Todd. *Collective Action: Theory and Applications*. Ann Arbor, MI: University of Michigan Press, 1992.

Sandler, Todd, and Tschirhart, John. "Club Theory: Thirty Years Later." *Public Choice* 93 (December, 1997): 335-355.

Schmalensee, Richard. *The Economics of Advertising*. Amsterdam: North-Holland, 1972.

Shapiro, Carl. "Advertising and Welfare: Comment." *Bell Journal of Economics* 11 (Autumn, 1980): 749-752.

Sterbenz, Frederic P., and Sandler, Todd. "Sharing among Clubs: A Club of Clubs Theory." *Oxford Economic Papers* 44 (January, 1992): 1-19.

Stigler, George J., and Becker, Gary S. "De Gustibus Non Est Disputandum." *American Economic Review* 67 (March, 1997): 76-90.

Stole, Lars. "Mechanism Design Under Common Agency." Mimeo, MIT, 1990.

Varian, Hal. "Sequential Contributions to Public Goods." *Journal of Public Economics* 53 (February 1994): 165-186.

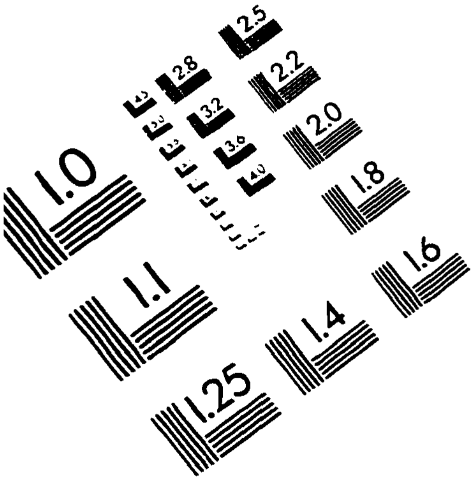
Varian, Hal R. *Microeconomic Analysis*, 3rd Ed. New York: W.W. Norton and Company, 1992.

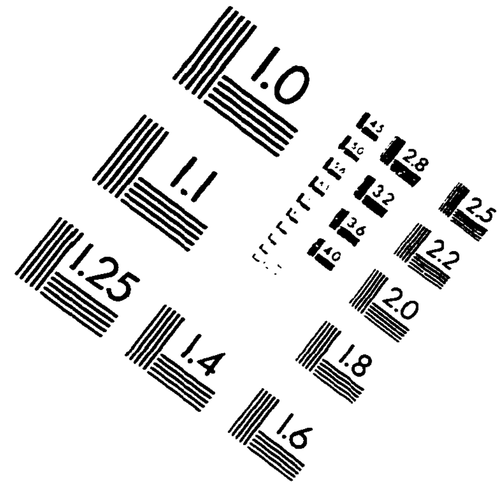
Wilson, James. *Bureaucracy: What Government Agencies Do and Why They Do It*. New York: Basic Books, 1989.

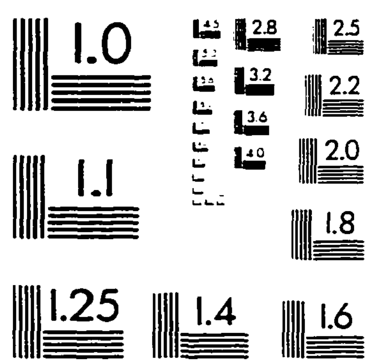
ACKNOWLEDGEMENTS

I would like to express my deepest appreciation to Dr. Todd Sandler, my major professor, for his help, encouragement, and unswerving support during my research and in the writing of this dissertation. I also would like to express my gratitude to Dr. Harvey Lapan and my thanks to the rest of my committee members, Dr. Herman Quirmbach, Dr. Herbert T. David, and Dr. Arne Hallam for their time. I also thank my family and friends, and especially Jeannie and Tristan, who sometimes seemed to have served as the sixth and seventh members of my committee.

IMAGE EVALUATION TEST TARGET (QA-3)

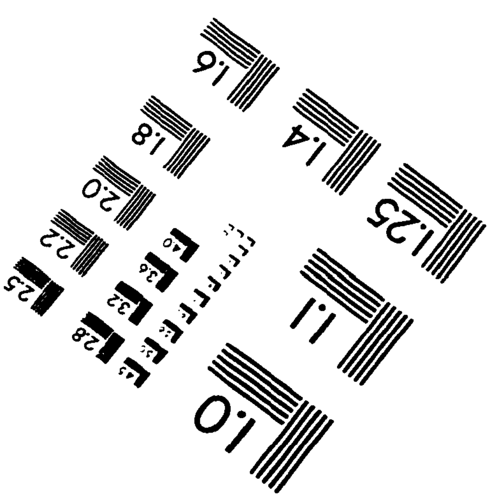


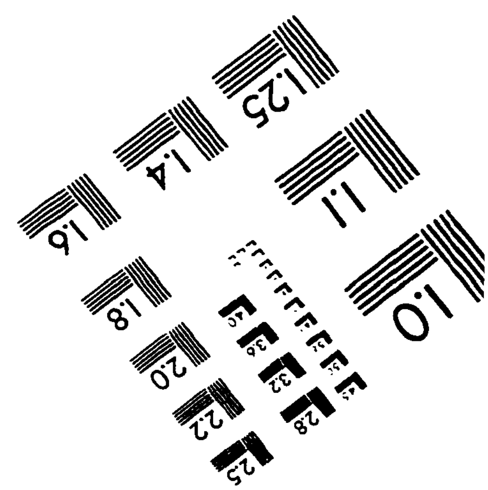




150mm

6″





APPLIED IMAGE . Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989